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Research Article

Beta type integral operator involving generalized Bessel-Maitland Function

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Abstract: The main object of the present paper is to establish integral involving generalized Bessel-Maitland functions $J_{\alpha,\beta,\nu,\sigma,\delta,p}^{\mu,\rho,\gamma,q}(z)$ defined by Khan et al. [6], which are expressed in the terms of generalized (Wright) hypergeometric functions. Some interesting special cases involving generalized Mittag-Leffler functions are deduced.

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1. Introduction

The special function of the form defined by the series representation

$$J_{\nu}^{\mu}(z)=\sum_{n=0}^{\infty}\frac{(-z)^{n}}{n!\Gamma(\mu n+\nu+1)}=\phi(\mu,\nu+1;-z), \tag{1.1}$$

is known as Bessel-Maitland function, or theWright generalized function (see [10;Eq.(8.3)]).It has a wide application in the problem of physics, chemistry, biology, engineering and applied sciences. The theory of Bessel functions is intimately connected with thetheory of certain types of differential equations. A detailed account of applications of Bessel functions are given in the book of Watson [17].

The generalized hypergeometric function represented as follows (see [20]):

$${}_{p}F_{q}\left[\begin{matrix}(\alpha_{p});\\(\beta_{q});\end{matrix}z\right]=\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{p}(\alpha_{j})_{n}z^{n}}{\prod_{j=1}^{q}(\beta_{j})_{n}n!}, \tag{1.2}$$

provided $p\leq q$; $p=q+1$ and $|z|<1$ and $(\alpha)_{n}$is well known Pochhammer symbol.

The Fox-Wright generalization $p\Psi q(z)$ of hypergeometric pFq function is given by (cf [13, 19, 20]):

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$$ {}_p\psi_q\left[\begin{matrix}(\alpha_1, A_1),....(\alpha_p, A_p);\\ (\beta_1, B_1),....(\beta_q, B_q);\end{matrix} z\right] = \sum_{n=0}^{\infty} \frac{\Gamma(\alpha_1 + A_1 n)...\Gamma(\alpha_p + A_p n) z^n}{\Gamma(\beta_1 + B_1 n)...\Gamma(\beta_q + B_q n) n!}, \tag{1.3}$$

where $A_j>0$ (j = 1, 2..., p) ;$B_j>0$ (j = 1, 2, ..., q) and $1 + \sum_{j=1}^{q} B_j - \sum_{j=1}^{p} A_j \geq 0$.

In recent years, several extension of some well-known special functions have been introduced by many authors, (see [1-6]).

Singh et al. [16] introduced the following generalization of Bessel-Maitland function:

$$ J_{\nu,q}^{\mu,\gamma}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{qn}(-z)^n}{n!\Gamma(\mu n + \nu + 1)}, \tag{1.4}$$

where μ, ν, γ $\in$C, $Re(\mu) \geq 0$, $Re(\nu) \geq -1$,$Re(\gamma) \geq 0$ and $q \in (0, 1)\cup$N and $(\gamma)_0= 1$, $(\gamma)_{qn}= \dfrac{\Gamma(\gamma + qn)}{\gamma}$ denotes the generalized Pochhammer symbol (see Rainville,[12]).

Recently, Ghayasuddin and Khan [1] introduced and investigated generalized Bessel-Maitland function defined as

$$ J_{\nu,\gamma,\delta}^{\mu,p,q}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{qn}(-z)^n}{\Gamma(\mu n + \nu + 1)(\delta)_{pn}}, \tag{1.5}$$

where μ, ν, γ, δ $\in$C, $\Re(\mu) \geq 0$,$\Re(\nu) \geq -1$,$\Re(\gamma) \geq 0$,$\Re(\delta) \geq 0$; p, q >0 and $q <\Re(\alpha) + p$.

In particular Khan *et al.* [6] introduced and investigated a new extension of Bessel-Maitland function as follows:

$$ J_{\alpha,\beta,\sigma,\nu,\delta,p}^{\mu,\rho,\gamma,q}(z) = \sum_{n=0}^{\infty} \frac{(\mu)_{\rho n}(\gamma)_{qn}(-z)^n}{\Gamma(n\beta + \alpha + 1)(\delta)_{pn}(\nu)_{n\sigma}}, \tag{1.6}$$

where α, β, μ, ρ, ν, γ, σ, δ $\in$C;$\Re(\alpha)$ >0,$\Re(\beta)$ >0,$\Re(\rho)$ >0,$\Re(\mu)$ >0,$\Re(\nu)$ >0,$\Re(\alpha) \geq -1$,$\Re(\gamma)$ >0,$\Re(\delta)$ >0,$\Re(\sigma)$ >0; p, q >0, and $q <\Re(\alpha) + p$.

2. Relation with Mittag-Leffler functions:

(i) On replacing α by $\alpha - 1$ in (1.6), we get the following interesting relation:

$$ J_{\alpha-1,\beta,\sigma,\nu,\delta,p}^{\mu,\rho,\gamma,q}(-z) = E_{\alpha,\beta,\sigma,\nu,\delta,p}^{\mu,\rho,\gamma,q}(z), \tag{1.7}$$

where $E_{\alpha,\beta,\sigma,\nu,\delta,p}^{\mu,\rho,\gamma,q}(z)$ is the Mittag-Leffler function defined by Khan and Ahmed [7].

(ii) On setting $\mu = \nu = \sigma = \rho = 1$ and replacing α by $\alpha - 1$ in (1.6), we get

$$ J_{\alpha-1,\beta,1,1,\delta,p}^{1,1,\gamma,q}(-z) = E_{\alpha,\beta,p}^{\delta,\gamma,q}(z) \tag{1.8}$$

where $E_{\alpha,\beta,p}^{\delta,\gamma,q}(z)$ is the Mittag-Leffler function defined by Salim and Faraz [14].

(iii) On setting $\mu = \nu = \sigma = \rho = \delta = p = 1$ and replacing α by $\alpha - 1$ in (1.6), we get

$$ J_{\alpha-1,\beta,1,1,1,1}^{1,1,\gamma,q}(-z) = E_{\alpha,\beta}^{\gamma,q}(z) \tag{1.9}$$

where $E_{\alpha,\beta}^{\gamma,q}(z)$ is the Mittag-Leffler function defined by Shukla and Prajapati [15].

(iv) On setting $\mu = \nu = \sigma = \rho = \delta = p = q = 1$ and replacing α by $\alpha-1$ in (1.6), we get

$$ J_{\alpha-1,\beta,1,1,1,1}^{1,1,\gamma,1}(-z) = E_{\alpha,\beta}^{\gamma}(z) \qquad 1.10)$$

where $E_{\alpha,\beta}^{\gamma}(z)$ is the Mittag-Leffler function defined by Prabhakar [11].

(v) On setting $\mu = \nu = \sigma = \rho = \delta = \gamma = p = q = 1$ and replacing α by $\alpha-1$ in (1.6), we get

$$J_{\alpha-1,\beta,1,1,1,1}^{1,1,1,1}(-z)=E_{\alpha,\beta}(z) \tag{1.11}$$

where $E_{\alpha,\beta}(z)$ is the Mittag-Leffler function defined by Wiman [18].

(vi) On setting $\mu=\nu=\sigma=\rho=\delta=\gamma=p=q=1$, $\alpha=0$ and replacing α by $\alpha-1$ in (1.6), we get

$$J_{0,\beta,1,1,1,1}^{1,1,1,1}(-z)=E_{\beta}(z) \tag{1.12}$$

where $E_{\beta}(z)$ is the Mittag-Leffler function defined by GhostaMittag-Leffler [9].

For our present investigation, the following interesting and useful result due to MacRobert [8] will be required:

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}[ax+b(1-x)]^{-\alpha-\beta}dx=\frac{\Gamma\ \alpha\ \Gamma\ \beta}{a^{\alpha}b^{\beta}\Gamma(\alpha+\beta)}, \tag{1.13}$$

provided $\Re(\alpha)>0$ and $\Re(\beta)>0$.

3. Beta type integral operator involving generalized Bessel-Maitland function

Theorem 2.1. If $\alpha, \beta, \gamma, \mu, \eta, \nu, \sigma, \rho, \delta, \lambda \in \mathbb{C}$, $\sigma+\eta+p-\rho-q>0$, $\Re(\alpha)>0, \Re(\beta)>0, \Re(\gamma)>0, \Re(\mu)>0, \Re(\lambda)\geq-1, \Re(\nu)>0, \Re(\delta)>0, \Re(\sigma)>0, \Re(\rho)>0, \Re(\eta)>0, p, q>0$ and $q<\Re(\alpha)+p$, then

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}[ax+b(1-x)]^{-\alpha-\beta}J_{\eta,\lambda,\sigma,\nu,\delta,p}^{\mu,\rho,\gamma,q}\left[\frac{2abx(1-x)}{[ax+b(1-x)]^2}\right]dx=$$

$$=\frac{\Gamma\ \nu\ \Gamma\ \delta}{a^{\alpha}b^{\beta}\Gamma\ \mu\ \Gamma\ \gamma}\,{}_5\psi_4\left[\begin{matrix}(\mu,\rho),(\gamma.q),(\alpha.1),(\beta,1),(1,1);\\(\nu,\sigma),(\delta,p),(\lambda+1,\eta),(\alpha+\beta,2);\end{matrix}\ -2\right]. \tag{2.1}$$

Proof. To establish our main result(2.1), we denote the left-hand side of (2.1) by $I$and then using (1.6), we have

$$I=\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}[ax+b(1-x)]^{-\alpha-\beta}J_{\eta,\lambda,\sigma,\nu,\delta,p}^{\mu,\rho,\gamma,q}(z)\left[\frac{2abx(1-x)}{[ax+b(1-x)]^2}\right]dx. \tag{2.2}$$

Now changing the order of integration and summation, which is seen to be justified due to the uniform convergence of the series in the interval (0,1), we arrive

$$I=\frac{\Gamma\ \nu\ \Gamma\ \delta}{a^{\alpha}b^{\beta}\Gamma\ \mu\ \Gamma\ \gamma}\sum_{m=0}^{\infty}\frac{\Gamma(\mu+\rho m)\Gamma(\gamma+qn)\Gamma(\alpha+m)\Gamma(\beta+m)\Gamma(m,1)}{\Gamma(\nu+\sigma m)\Gamma(\delta+pm)\Gamma(\lambda+1,\eta m)\Gamma(\alpha+\beta+2m)}\frac{-2^m}{m!}. \tag{2.3}$$

Finally, summing up the above series with the help of (1.3), we easily arrive at the right-hand side of (2.1). This completes the proof of our main result. Next, we consider other variation of (2.1). In fact, we establish an integral formula for the Bessel-Maitland function which is expressed in terms of the generalized hypergeometric function *pFq*.

Variation of (2.1): Let the conditions of our main result be satisfied, then the following integral formula holds true:

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}[ax+b(1-x)]^{-\alpha-\beta}J_{\eta,\lambda,\sigma,\nu,\delta,p}^{\mu,\rho,\gamma,q}\left[\frac{2abx(1-x)}{[ax+b(1-x)]^2}\right]dx=\frac{\Gamma\ \alpha\ \Gamma\ \beta}{a^{\alpha}b^{\beta}\Gamma(\lambda+1)\Gamma(\alpha+\beta)}$$

$$\times{}_{\rho+q+3}F_{\sigma+\eta+p+2}\left[\begin{matrix}\Delta(\rho;\mu),\Delta(q;\gamma),\alpha,\beta,1;\\\Delta(\sigma,\nu),\Delta(p;\delta),\Delta(\eta;\lambda+1),\Delta(2,\alpha+\beta);\end{matrix}\ \frac{-\rho^{\rho}q^{q}}{2\sigma^{\sigma}p^{p}\eta^{\eta}}\right]. \tag{2.4}$$

Proof: In order to prove the result (2.4), using the results

$$\Gamma(\alpha+n)=\Gamma(\alpha)(\alpha)_n \quad \text{and} \quad (l)_{kn}=k^{kn}\left(\frac{l}{k}\right)_n\left(\frac{l+1}{k}\right)_n\cdots\left(\frac{l+k-1}{k}\right)_n,$$

(Gauss multiplication theorem) in (2.3) and summing up the given series with the help of (1.2), we easily arrive at our required result (2.4).

4. Special Cases

(i). On replacing λ by $\lambda-1$ in (2.1) and then by using (1.7), we get:

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}[ax+b(1-x)]^{-\alpha-\beta}E^{\mu,\rho,\gamma,q}_{\eta,\lambda,\sigma,\nu,\delta,p}\left[\frac{2abx(1-x)}{[ax+b(1-x)]^2}\right]dx=$$

$$\frac{\Gamma\ \nu\ \Gamma\ \delta}{a^{\alpha}b^{\beta}\Gamma\ \mu\ \Gamma\ \gamma}\ {}_5\psi_4\left[\begin{matrix}(\mu,\rho),(\gamma.q),(\alpha.1),(\beta,1),(1,1);\\(\nu,\sigma),(\delta,p),(\lambda+1,\eta),(\alpha+\beta,2);\end{matrix}\ 2\right]. \quad (3.1)$$

where $\alpha, \beta, \gamma, \mu, \eta, \nu, \sigma, \rho, \delta, \lambda \in \mathbb{C}$, $\Re(\alpha)>0, \Re(\beta)>0, \Re(\gamma)>0, \Re(\mu)>0, \Re(\nu)>0, \Re(\lambda)>0, \Re(\delta)>0, \Re(\sigma)>0, \Re(\rho)>0, \Re(\eta)>0$, $p, q>0$ and $q<\Re(\alpha)+p$.

(ii). On replacing λ by $\lambda-1$ in (2.4) and then by using (1.7), we find:

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}[ax+b(1-x)]^{-\alpha-\beta}E^{\mu,\rho,\gamma,q}_{\eta,\lambda,\sigma,\nu,\delta,p}\left[\frac{2abx(1-x)}{[ax+b(1-x)]^2}\right]dx=\frac{\Gamma\ \alpha\ \Gamma\ \beta}{a^{\alpha}b^{\beta}\Gamma\ \lambda\ \Gamma(\alpha+\beta)}$$

$$\times{}_{\rho+q+3}F_{\sigma+\eta+p+2}\left[\begin{matrix}\Delta(\rho;\mu),\Delta(q;\gamma),\alpha,\beta,1;\\\Delta(\sigma,\nu),\Delta(p;\delta),\Delta(\eta;\lambda+1),\Delta(2,\alpha+\beta);\end{matrix}\ \frac{\rho^{\rho}q^{q}}{2\sigma^{\sigma}p^{p}\eta^{\eta}}\right]. (3.2)$$

where $\alpha, \beta, \gamma, \mu, \eta, \nu, \sigma, \rho, \delta, \lambda \in \mathbb{C}$, $\Re(\alpha)>0, \Re(\beta)>0, \Re(\gamma)>0, \Re(\mu)>0, \Re(\nu)>0, \Re(\lambda)>0, \Re(\delta)>0, \Re(\sigma)>0, \Re(\rho)>0, \Re(\eta)>0$, $p, q>0$ and $q<\Re(\alpha)+p$.

(iii). On setting $\mu=\nu=\rho=\sigma=1$ and replacing λ by $\lambda-1$ in (2.1) and then by using (1.8), we attain:

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}[ax+b(1-x)]^{-\alpha-\beta}E^{\gamma,\delta,q}_{\eta,\lambda,p}\left[\frac{2abx(1-x)}{[ax+b(1-x)]^2}\right]dx=$$

$$\frac{\Gamma\ \delta}{a^{\alpha}b^{\beta}\Gamma\ \gamma}\ {}_4\psi_3\left[\begin{matrix}(\gamma.q),(\alpha.1),(\beta,1),(1,1);\\(\delta,p),(\lambda,\eta),(\alpha+\beta,2);\end{matrix}\ 2\right]. \quad (3.3)$$

where $\alpha, \beta, \gamma, \eta, \delta, \lambda \in \mathbb{C}$, $\Re(\alpha)>0, \Re(\beta)>0, \Re(\gamma)>0, , \Re(\lambda)>0, \Re(\delta)>0, \Re(\eta)>0$, $p, q>0$ and $q<\Re(\alpha)+p$.

(iv). On setting $\mu=\nu=\rho=\sigma=1$ and replacing λ by $\lambda-1$ in (2.4) and then by using (1.8), we attain:

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}[ax+b(1-x)]^{-\alpha-\beta}E^{\gamma,\delta,q}_{\eta,\lambda,p}\left[\frac{2abx(1-x)}{[ax+b(1-x)]^2}\right]dx=\frac{\Gamma\ \alpha\ \Gamma\ \beta}{a^{\alpha}b^{\beta}\Gamma\ \lambda\ \Gamma(\alpha+\beta)}$$

$$\times{}_{q+3}F_{\eta+p+2}\left[\begin{matrix}\Delta(q;\gamma),\alpha,\beta,1;\\\Delta(p;\delta),\Delta(\eta;\lambda),\Delta(2,\alpha+\beta);\end{matrix}\ \frac{q^{q}}{2p^{p}\eta^{\eta}}\right]. (3.4)$$

where $\alpha, \beta, \gamma, \eta, \delta, \lambda \in \mathbb{C}$, $\Re(\alpha)>0, \Re(\beta)>0, \Re(\gamma)>0, , \Re(\lambda)>0, \Re(\delta)>0, \Re(\eta)>0$, $p, q>0$ and $q<\Re(\alpha)+p$.

(v) On setting $\mu=\nu=\rho=\sigma=p=\delta=1$ and replacing λ by $\lambda-1$ in (2.1) and then by using (1.9), we attain:

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}[ax+b(1-x)]^{-\alpha-\beta}E^{\gamma,q}_{\eta,\lambda}\left[\frac{2abx(1-x)}{[ax+b(1-x)]^2}\right]dx=$$

$$=\frac{1}{a^{\alpha}b^{\beta}\Gamma(\gamma)}\,{}_4\psi_2\left[\begin{matrix}(\gamma,q),(\alpha,1),(\beta,1),(1,1);\\(\lambda,\eta),(\alpha+\beta,2);\end{matrix}\,2\right]. \quad (3.5)$$

where $\alpha, \beta, \gamma, \eta, \lambda \in C$, $\Re(\alpha)>0, \Re(\beta)>0, \Re(\gamma)>0, \Re(\lambda)>0, \Re(\eta)>0$, $q \in (0, 1)\cup N$.

(vi) On setting $\mu = v = \rho = \sigma = p = \delta = 1$ and replacing λ by $\lambda - 1$ in (2.4) and then by using (1.9), we attain:

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}[ax+b(1-x)]^{-\alpha-\beta} E_{\eta,\lambda}^{\gamma,q}\left[\frac{2abx(1-x)}{[ax+b(1-x)]^2}\right]dx=\frac{\Gamma(\alpha)\Gamma(\beta)}{a^{\alpha}b^{\beta}\Gamma(\lambda)\Gamma(\alpha+\beta)}$$

$$\times{}_{q+3}F_{\eta+2}\left[\begin{matrix}\Delta(q;\gamma),\alpha,\beta,1;\\\Delta(\eta;\lambda),\Delta(2,\alpha+\beta);\end{matrix}\,\frac{q^q}{2\eta^{\eta}}\right] \quad (3.6)$$

where $\alpha, \beta, \gamma, \eta, \lambda \in C$, $\Re(\alpha)>0, \Re(\beta)>0, \Re(\gamma)>0, \Re(\lambda)>0, \Re(\eta)>0$, $q \in (0, 1)\cup N$.

(vii) On setting $\mu = v = \rho = \sigma = p = q = \delta = 1$ and replacing λ by $\lambda - 1$ in (2.1) and then by using (1.10), we attain:

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}[ax+b(1-x)]^{-\alpha-\beta} E_{\eta,\lambda}^{\gamma}\left[\frac{2abx(1-x)}{[ax+b(1-x)]^2}\right]dx=$$

$$=\frac{1}{a^{\alpha}b^{\beta}\Gamma(\gamma)}\,{}_4\psi_2\left[\begin{matrix}(\gamma,1),(\alpha,1),(\beta,1),(1,1);\\(\lambda,\eta),(\alpha+\beta,2);\end{matrix}\,2\right]. \quad (3.7)$$

where $\alpha, \beta, \gamma, \eta, \lambda \in C$, $\Re(\alpha)>0, \Re(\beta)>0, \Re(\gamma)>0, \Re(\lambda)>0, \Re(\eta)>0$.

(viii) On setting $\mu = v = \rho = \sigma = p = \delta = 1$ and replacing λ by $\lambda - 1$ in (2.4) and then by using (1.10), we attain:

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}[ax+b(1-x)]^{-\alpha-\beta} E_{\lambda,\delta}^{\gamma}\left[\frac{2abx(1-x)}{[ax+b(1-x)]^2}\right]dx=\frac{\Gamma(\alpha)\Gamma(\beta)}{a^{\alpha}b^{\beta}\Gamma(\lambda)\Gamma(\alpha+\beta)}$$

$$\times{}_4F_{\eta+2}\left[\begin{matrix}\gamma,\alpha,\beta,1;\\\Delta(\eta;\lambda),\Delta(2,\alpha+\beta);\end{matrix}\,\frac{1}{2\eta^{\eta}}\right]. \quad (3.8)$$

where $\alpha, \beta, \gamma, \eta, \lambda \in C$, $\Re(\alpha)>0, \Re(\beta)>0, \Re(\gamma)>0, \Re(\lambda)>0, \Re(\eta)>0$.

(ix) On setting $\mu = v = \rho = \sigma = p = q = \delta = \gamma = 1$ and replacing λ by $\lambda - 1$ in (2.1) and then by using (1.11), we attain:

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}[ax+b(1-x)]^{-\alpha-\beta} E_{\eta,\lambda}\left[\frac{2abx(1-x)}{[ax+b(1-x)]^2}\right]dx=$$

$$=\frac{1}{a^{\alpha}b^{\beta}}\,{}_3\psi_2\left[\begin{matrix}(\alpha,1),(\beta,1),(1,1);\\(\lambda,\eta),(\alpha+\beta,2);\end{matrix}\,2\right]. \quad (3.9)$$

where $\alpha, \beta, \eta, \lambda \in C$; $\Re(\alpha)>0, \Re(\beta)>0, \Re(\lambda)>0, \Re(\eta)>0$.

(x) On setting $\mu = v = \rho = \sigma = p = q = \delta = \gamma = 1$ and replacing λ by $\lambda-1$ in (2.4)and then by using (1.11), we attain:

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}[ax+b(1-x)]^{-\alpha-\beta} E_{\eta,\lambda}\left[\frac{2abx(1-x)}{[ax+b(1-x)]^2}\right]dx=\frac{\Gamma(\alpha)\Gamma(\beta)}{a^{\alpha}b^{\beta}\Gamma(\lambda)\Gamma(\alpha+\beta)}$$

$$\times{}_3F_{\eta+2}\left[\begin{matrix}\alpha,\beta,1;\\\Delta(\eta;\lambda),\Delta(2,\alpha+\beta);\end{matrix}\,\frac{1}{2\eta^{\eta}}\right] \quad (3.10)$$

where $\alpha, \beta, \eta, \lambda \in C$; $\Re(\alpha)>0, \Re(\beta)>0, \Re(\lambda)>0, \Re(\eta)>0$.

(xi) On setting $\mu = v = \rho = \sigma = p = q = \delta = \gamma = 1$ and $\lambda = 0$ in (2.1) and then by using (1.12), we get

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}[ax+b(1-x)]^{-\alpha-\beta} E_{\eta}\left[\frac{2abx(1-x)}{[ax+b(1-x)]^2}\right]dx=$$

$$= \frac{1}{a^{\alpha} b^{\beta}} \, {}_3\psi_2 \left[\begin{matrix} (\alpha.1),(\beta,1),(1,1); \\ \eta,(\alpha+\beta,2); \end{matrix} 2 \right]. \tag{3.11}$$

where $\alpha, \beta, \eta \in \mathbb{C}; \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\eta) > 0$.

(xii). On setting $\mu = \nu = \rho = \sigma = p = q = \delta = \gamma = 1$ and $\lambda = 0$ in (2.4) and then by using (1.12), we attain:

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}[ax+b(1-x)]^{-\alpha-\beta} E_{\eta} \left[\frac{2abx(1-x)}{[ax+b(1-x)]^2} \right] dx = \frac{\Gamma\ \alpha\ \Gamma\ \beta}{a^{\alpha} b^{\beta} \Gamma(\alpha+\beta)}$$

$$\times {}_3F_{\eta+2} \left[\begin{matrix} \alpha, \beta, 1; \\ \Delta(\eta;\lambda), \Delta(2,\alpha+\beta); \end{matrix} \frac{1}{2\eta^{\eta}} \right]. \tag{3.12}$$

where $\alpha, \beta, \eta \in \mathbb{C}; \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\eta) > 0$.

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