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Research Article

4-TOTAL EDGE PRODUCT CORDIAL LABELING OF SOME STANDARD GRAPHS

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Abstract: In this paper, we will determine the 4-total edge product cordial (4-TEPC) labeling of certain classes of graphs namely wheel, gear and helm graphs.

***Keywords**:* 4-TEPC labeling, wheel, gear and helm graphs.

1. INTRODUCTION AND PRELIMINARIES

We begin with finite, undirected, simple and connected graph $\mathcal{G} = (V(\mathcal{G}), E(\mathcal{G}))$. The set $V(\mathcal{G})$ is called vertex set and the set $E(\mathcal{G})$ is called edge set of graph $\mathcal{G}$. Order of a graph is the number of vertices in G and size of a graph is the number of edges in G. We follow the standard notations and terminology of graph theory as in [1]. Graph labeling were first introduced in the late 1960s. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling). We have the following notations, in order to know cordial labeling f and its sorts.

(1) The number of vertices labeled by x is $v_f(x)$;

(2) The number of edges labeled by x is $e_f(x)$;

(3) $v_f(x,y) = v_f(x) - v_f(y)$;

(4) $e_f(x,y) = e_f(x) - e_f(y)$;

(5) $sum(x) = v_f(x) + v_f(x)$;

(6) $\mathbb{Z}_k$ denotes the first k non negative integers, i.e $\mathbb{Z}_k = \{0,1,2,\dots,k-1\}$.

Cordial labeling was introduced by Cahit (see [2]). Now we will define cordial labeling and its different types.

Definitions 1.1. (1) Let $f: V(\mathcal{G}) \to \mathbb{Z}_2$ be a mapping that induces $f^*: E(\mathcal{G}) \to \mathbb{Z}_2$ as $f^*(uv) = |f(u) - f(v)|$ where $uv \in E(\mathcal{G})$. Then f is called cordial labeling if $|v_f(0,1)| \leq 1$ and $|e_f(0,1)| \leq 1$.

(2) Let $f: V(\mathcal{G}) \to \mathbb{Z}_2$ be a mapping that induces $f^*: E(\mathcal{G}) \to \mathbb{Z}_2$ as $f^*(uv) = f(u)f(v)$ where $uv \in E(\mathcal{G})$. Then f is called product cordial labeling if $|v_f(0,1)| \leq 1$ and $|e_f(0,1)| \leq 1$. For details see [3].

(3) Let $f: V(\mathcal{G}) \to \mathbb{Z}_2$ be a mapping that induces $f^*: E(\mathcal{G}) \to \mathbb{Z}_2$ as $f^*(uv) = f(u)f(v)$ where $uv \in E(\mathcal{G})$ Then f is called total product cordial labeling if $|sum(0) - sum(1)| \leq 1$. For details see [4, 5].

(4) Let $f: V(\mathcal{G}) \to \mathbb{Z}_k$, $2 \leq k \leq |E(G)|$ be a mapping that induces $f^*: E(\mathcal{G}) \to \mathbb{Z}_k$ as $f^*(uv) = f(u)f(v)$ (modk) where $uv \in E(\mathcal{G})$. Then f is called a k-total product cordial labeling if $|sum(a) - sum(b)| \leq 1$ for all $a, b \in \mathbb{Z}_k$. For details see [6].

(5) Let $f: V(\mathcal{G}) \rightarrow \mathbb{Z}_2$ be a mapping that induces $f^*: E(\mathcal{G}) \rightarrow \mathbb{Z}_2$ such that $f^*(u) = f(e_1)f(e_2) \dots f(e_n)$ for edges $e_1, e_2, \dots, e_n$ incident to u, then f is called edge product cordial labeling if $|v_f(0,1)| \leq 1$ and $|e_f(0,1)| \leq 1$. For details see [7,8].

(6) Let $f: V(\mathcal{G}) \rightarrow \mathbb{Z}_2$ be a mapping that induces $f^*: V(\mathcal{G}) \rightarrow \mathbb{Z}_2$ such that $f^*(u) = f(e_1)f(e_2) \dots f(e_n)$ for edges $e_1, e_2, \dots, e_n$ that are incident to u, then f is called a total edge product cordial labeling if $|sum(0) - sum(1)| \leq 1$. For details see [9,10].

(7) Let $f: V(\mathcal{G}) \rightarrow \mathbb{Z}_k$, $2 \leq k \leq |E(G)|$ be a mapping that induces $f^*: V(\mathcal{G}) \rightarrow \mathbb{Z}_k$ such that $f^*(u) = f(e_1)f(e_2) \dots f(e_n)$ (mod k) for edges $e_1, e_2, \dots, e_n$ incident to u, then f is called k-total edge product cordial labeling if it satisfy $|sum(a) - sum(b)| \leq 1$ for all $a, b \in \mathbb{Z}_k$. For details see [11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. Now we will define the different family of graphs.

Definitions 1.2. (1) We obtain a wheel graph W_n when we add an addition as vertex to a cycle C_n for $n \geq 3$ and connect this new vertex to each of n vertices of C_n by new edges. See Figure 1 for the example of wheel graph W_6.

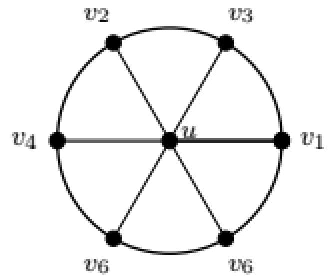

Figure 1. Wheel graph W_6

(2) A gear graph G_n is the graph obtained by adding a vertex between all pair of adjacent vertices of the cycle C_n in the wheel graph. See Figure 2 for the example of gear graph G_6.

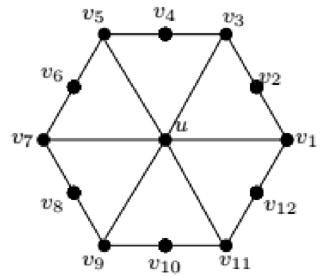

Figure 2. Gear graph G6

(3) The helm graph H_n is the graph obtained from a wheel graph by adjoining a pendent edge at each vertex of the cycle C_n . See Figure 3 for the example of helm graph H_6.

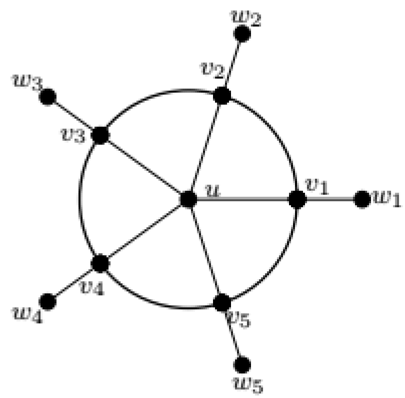

Figure 3. Helm graph H_6

2. MAIN RESULTS

In this Section, we discussed 4-total edge product cordial (4-TEPC) labeling of some graphs.

2.1 4-TEPC labeling of Wheel graph

Theorem 2.1. Let $\mathcal{G}$ be a wheel graph W_n then $\mathcal{G}$ admits 4-TEPC labeling.

Proof. Let $V(\mathcal{G}) = \{u, v_x,\ 1 \le x \le n\}$ and $E(\mathcal{G}) = \{uv_x,\ 1 \le x \le n\} \cup \{v_x v_{x+1},\ 1 \le x \le n-1\} \cup \{v_n v_1\}$ be the vertex-set and edge-set of the wheel graph respectively as shown in Figure 1. Now we have the following four cases:

Case 1: Let n ≡ 0 (mod 4) which implies $n = 4t$, for any integer t ≥ 1. We define $f: E(\mathcal{G}) \to \mathbb{Z}_4$ as

$$f(uv_x) = \begin{cases} 0, & if \quad 1 \le x \le t; \\ 2, & if \quad t+1 \le x \le 2t; \\ 1, & if \quad 2t+1 \le x \le 4t. \end{cases}$$

$$f(v_x v_{x+1}) = \begin{cases} 3, & if \quad 1 \le x \le 3t; \\ 2, & if \quad 3t+1 \le x \le 4t-1. \end{cases} \quad \text{and} \quad f(v_{4t} v_1) = 2.$$

So we obtain $sum(0) = sum(1) = sum(3) = 3t,\ sum(2) = 3t + 1$. Thus $|sum(x) - sum(y)| \le 1$ for all $x \ne y \in \mathbb{Z}_4$. Consequently f is 4-TEPC labeling.

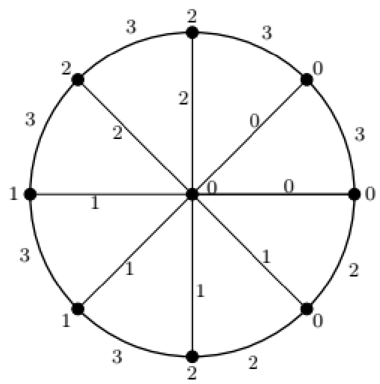

FIGURE 4. 4-TEPC labeling of W_8

Case 2: Let n ≡ 1 (mod 4) which implies $n = 4t + 1$, for t ≥ 1.

Case 2.1: If $t = 1$ then we define $f: E(\mathcal{G}) \to \mathbb{Z}_4$ as

$$f(uv_x) = \begin{cases} 0, & if \quad 1 \le x \le t; \\ 2, & if \quad t+1 \le x \le 3t; \\ 1, & if \quad 3t+1 \le x \le 4t+1. \end{cases}$$

$$f(v_x v_{x+1}) = \begin{cases} 2, & if \quad 1 \le x \le t; \\ 3, & if \quad t+1 \le x \le 3t+1. \end{cases} \quad \text{and} \quad f(v_{4t+1} v_1) = 3.$$

So we obtain $sum(0) = sum(1) = sum(2) = sum(3) = 3t + 1$. Thus $|sum(x) - sum(y)| \le 1$ for all $x \ne y \in \mathbb{Z}_4$. Consequently f is 4-TEPC labeling.

Case 2.2: If $t \ge 2$, then we have two subcases:

Case 2.2.1: When t is odd, we define $f: E(\mathcal{G}) \to \mathbb{Z}_4$ as

$$f(uv_x) = \begin{cases} 0, & if \quad 1 \le x \le \frac{3t-1}{2}; \\ 2, & if \quad \frac{3t+2}{2} \le x \le 3t; \\ 1, & if \quad 3t+1 \le x \le 4t+1. \end{cases}$$

$$f(v_x v_{x+1}) = \begin{cases} 1, & if \quad 2 \le x \le t; \\ 3, & if \quad t+1 \le x \le 4t. \end{cases},\ f(v_1 v_2) = 0 \ \text{and}\ f(v_{4t+1} v_1) = 3.$$

So we obtain $sum(0) = sum(1) = sum(2) = sum(3) = 3t + 1$. Thus $|sum(x) - sum(y)| \le 1$ for all $x \ne y \in \mathbb{Z}_4$. Consequently f is 4-TEPC labeling.

Case 2.2.2: When t is even, we define $f: E(\mathcal{G}) \to \mathbb{Z}_4$as

$$f(uv_x) = \begin{cases} 0, & if \quad 1 \le x \le \frac{3t}{2}; \\ 2, & if \quad \frac{3t+2}{2} \le x \le 3t; \\ 1, & if \quad 3t+1 \le x \le 4t+1. \end{cases}$$

$f(v_x v_{x+1}) = \begin{cases} 1, & if \quad 2 \le x \le t; \\ 3, & if \quad t+1 \le x \le 4t. \end{cases}$, $f(v_1 v_2) = 2$ and $f(v_{4t+1} v_1) = 3$.

So we obtain $sum(0) = sum(1) = sum(2) = sum(3) = 3t+1$. Thus $|sum(x) - sum(y)| \le 1$ for all $x \neq y \in \mathbb{Z}_4$. Consequently f is 4-TEPC labeling.

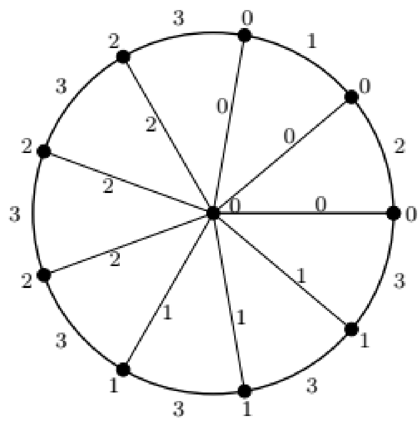

FIGURE 5. 4-TEPC labeling of W_9

Case 3: Let $n \equiv 2 \pmod 4$ which implies $n = 4t+2$, for any integer t ≥ 1.

Case 3.1: When t is odd, we define $f: E(\mathcal{G}) \to \mathbb{Z}_4$ as

$$f(uv_x) = \begin{cases} 0, & if \quad 1 \le x \le \frac{3t+1}{2}; \\ 2, & if \quad \frac{3t+3}{2} \le x \le 3t+1; \\ 1, & if \quad 3t+2 \le x \le 4t+2. \end{cases}$$

$f(v_x v_{x+1}) = \begin{cases} 1, & if \quad 1 \le x \le t; \\ 3, & if \quad t+1 \le x \le 4t+1. \end{cases}$ and $f(v_{4t+2} v_1) = 3$.

So we obtain $sum(0) = sum(1) = sum(3) = 3t+2$, $sum(2) = 3t+1$. Thus $|sum(x) - sum(y)| \le 1$ for all $x \neq y \in \mathbb{Z}_4$. Consequently f is 4-TEPC labeling.

Case 3.2: When t is even, we define $f: E(\mathcal{G}) \to \mathbb{Z}_4$ as

$$f(uv_x) = \begin{cases} 0, & if \quad 1 \le x \le \frac{3t}{2}; \\ 2, & if \quad \frac{3t+2}{2} \le x \le 3t+1; \\ 1, & if \quad 3t+2 \le x \le 4t+2. \end{cases}$$

$f(v_x v_{x+1}) = \begin{cases} 1, & if \quad 1 \le x \le t; \\ 3, & if \quad t+1 \le x \le 4t+1. \end{cases}$ and $f(v_{4t+2} v_1) = 3$.

So we obtain $sum(0) = 3t+1$, $sum(1) = sum(2) = sum(3) = 3t+2$. Thus $|sum(x) - sum(y)| \le 1$ for all $x \neq y \in \mathbb{Z}_4$. Consequently f is 4-TEPC labeling.

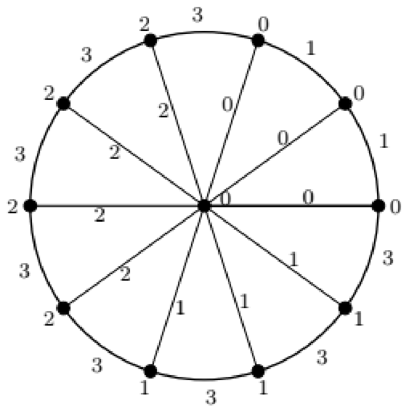

FIGURE 6. 4-TEPC labeling of W_{10}

Case 4: Let $n \equiv 3 \pmod 4$ which implies $n = 4t + 3$, for any integer $t \geq 1$.

Case 4.1: When t is odd, we define $f: E(\mathcal{G}) \to \mathbb{Z}_4$ as:

$$f(uv_x) = \begin{cases} 0, & if \quad 1 \leq x \leq \dfrac{3t+1}{2}; \\ 2, & if \quad \dfrac{3t+3}{2} \leq x \leq 3t+2; \\ 1, & if \quad 3t+3 \leq x \leq 4t+3. \end{cases}$$

$$f(v_x v_{x+1}) = \begin{cases} 1, & if \quad 1 \leq x \leq t; \\ 3, & if \quad t+1 \leq x \leq 4t+2. \end{cases} \text{ and } f(v_{4t+3}v_1) = 3.$$

So we obtain $sum(0) = sum(1) = 3t + 2$, $sum(3) = sum(2) = 3t + 3$. Thus $|sum(x) - sum(y)| \leq 1$ for all $x \neq y \in \mathbb{Z}_4$. Consequently f is 4-TEPC labeling.

Case 4.2: When t is even, we define $f: E(\mathcal{G}) \to \mathbb{Z}_4$ as

$$f(uv_x) = \begin{cases} 0, & if \quad 1 \leq x \leq \dfrac{3t+2}{2}; \\ 2, & if \quad \dfrac{3t+4}{2} \leq x \leq 3t+2; \\ 1, & if \quad 3t+3 \leq x \leq 4t+3. \end{cases}$$

$$f(v_x v_{x+1}) = \begin{cases} 1, & if \quad 1 \leq x \leq t+1; \\ 3, & if \quad t+2 \leq x \leq 4t+2. \end{cases} \text{ and } f(v_{4t+3}v_1) = 3.$$

So we obtain $sum(0) = sum(1) = 3t + 3$, $sum(2) = sum(3) = 3t + 2$. Thus $|sum(x) - sum(y)| \leq 1$ for all $x \neq y \in \mathbb{Z}_4$. Consequently f is 4-TEPC labeling.

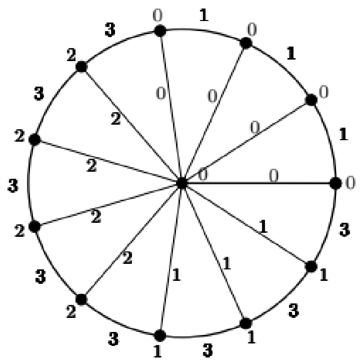

FIGURE 7. 4-TEPC labeling of W_{11}

2.2. 4-TEPC labeling of Gear graph

Theorem 2.2. Let $\mathcal{G}$ be a gear graph $\mathcal{G}_n$ then $\mathcal{G}$ admits 4-TEPC labeling.

Proof. Let $V(\mathcal{G}) = \{uv_x,\ 1 \le x \le n\}$ and $E(\mathcal{G}) = \{uv_{2x-1},\ 1 \le x \le n\} \cup \{v_x v_{x+1},\ 1 \le x \le 2n-1\} \cup \{v_{2n-1}v_1\}$ be the vertex-set and edge-set of the wheel graph respectively as shown in Figure 2. Now we have the following four cases:

Case 1: Let $n \equiv 0 \pmod 4$ which implies $n = 4t$, for any integer t ≥ 1. *We* define $f: E(\mathcal{G}) \to \mathbb{Z}_4$ as

$$f(uv_x) = \begin{cases} 0, & if \quad 1 \le x \le t; \\ 2, & if \quad t+1 \le x \le 2t; \\ 1, & if \quad 2t+1 \le x \le 4t. \end{cases}$$

$$f(v_x v_{x+1}) = \begin{cases} 3, & if \quad 1 \le x \le 5t; \\ 2, & if \quad 5t+1 \le x \le 8t-1. \end{cases} \quad \text{and} \quad f(v_{4t}v_1) = 2.$$

So we obtain $sum(0) = sum(1) = sum(3) = 5t,\ sum(2) = 5t+1$. Thus $|sum(x) - sum(y)| \le 1$ for all $x \ne y \in \mathbb{Z}_4$. Consequently f is 4-TEPC labeling.

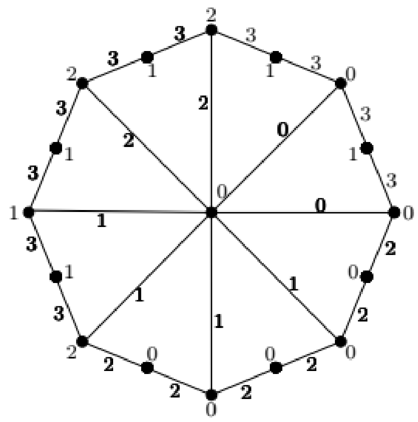

FIGURE 8. 4-TEPC labeling of $\mathcal{G}_8$

Case 2: Let $n \equiv 1 \pmod 4$ which implies $n = 4t+1$, for t ≥ 1.

Case 2.1: If $t = 1$ then we define $f: E(\mathcal{G}) \to \mathbb{Z}_4$ as

$$f(uv_x) = \begin{cases} 0, & if \quad 1 \le x \le t; \\ 2, & if \quad t+1 \le x \le 2t; \\ 1, & if \quad 2t+1 \le x \le 4t+1. \end{cases}$$

$$f(v_x v_{x+1}) = \begin{cases} 3, & if \quad 2 \le x \le 6t+1; \\ 2, & if \quad 6t+2 \le x \le 8t+1. \end{cases}, f(v_1v_2) = 0 \quad \text{and} \quad f(v_{4t+1}v_1) = 3.$$

So we obtain $sum(0) = sum(2) = 5t+1,\ sum(1) = sum(3) = 5t+2$. Thus $|sum(x) - sum(y)| \le 1$ for all $x \ne y \in \mathbb{Z}_4$. Consequently f is 4-TEPC labeling.

Case 2.2: If $t \ge 2$, then we have two subcases:

Case 2.2.1: When t is odd, we define $f: E(\mathcal{G}) \to \mathbb{Z}_4$ as

$$f(uv_x) = \begin{cases} 0, & if \quad 1 \le x \le \frac{3t-1}{2}; \\ 2, & if \quad \frac{3t+1}{2} \le x \le 3t-1; \\ 1, & if \quad 3t \le x \le 4t+1. \end{cases}$$

$$f(v_x v_{x+1}) = \begin{cases} 1, & if \quad 2 \le x \le t; \\ 3, & if \quad t+1 \le x \le 6t+1; \\ 2, & if \quad 6t+2 \le x \le 8t+1. \end{cases}, f(v_1v_2) = 0 \text{ and } f(v_{4t+1}v_1) = 1.$$

So we obtain $sum(0) = sum(2) = 5t+1,\ sum(1) = sum(3) = 5t+2$. Thus $|sum(x) - sum(y)| \le 1$ for all $x \ne y \in \mathbb{Z}_4$. Consequently f is 4-TEPC labeling.

Case 2.2.2: When t is even, we define $f: E(\mathcal{G}) \to \mathbb{Z}_4$as

$$f(uv_x)=\begin{cases}0, & if \quad 1\le x\le \frac{3t}{2};\\ 2, & if \quad \frac{3t+2}{2}\le x\le 3t;\\ 1, & if \quad 3t+1\le x\le 4t+1.\end{cases}$$

$$f(v_xv_{x+1})=\begin{cases}1, & if \quad 2\le x\le t;\\ 3, & if \quad t+1\le x\le 6t+2;\\ 2, & if \quad 6t+3\le x\le 8t+1.\end{cases},\ f(v_1v_2)=2 \text{ and } f(v_{4t+1}v_1)=0.$$

So we obtain $sum(0)=sum(1)=5t+1$, $sum(2)=sum(3)=5t+2$. Thus $|sum(x)-sum(y)|\le 1$ for all $x\neq y\in\mathbb{Z}_4$. Consequently f is 4-TEPC labeling.

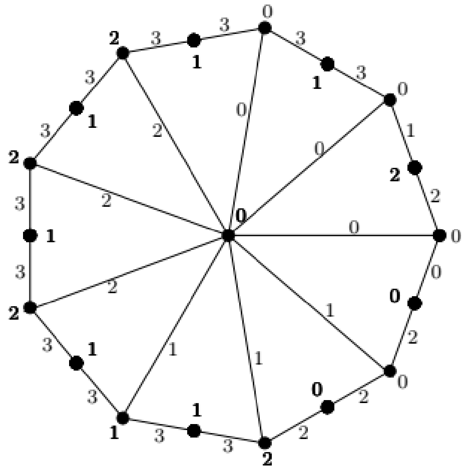

FIGURE 9. 4-TEPC labeling of $\mathcal{G}_9$

Case 3: Let $n\equiv 2 \pmod 4$ which implies $n=4t+2$, for any integer $t\ge 1$.

Case 3.1: When t is odd, we define $f: E(\mathcal{G})\to\mathbb{Z}_4$ as

$$f(uv_x)=\begin{cases}0, & if \quad 1\le x\le \frac{3t-1}{2};\\ 2, & if \quad \frac{3t+1}{2}\le x\le 3t-1;\\ 1, & if \quad 3t\le x\le 4t+2.\end{cases}$$

$$f(v_xv_{x+1})=\begin{cases}1, & if \quad 2\le x\le t;\\ 3, & if \quad t+1\le x\le 6t+1;\\ 2, & if \quad 6t+2\le x\le 8t+3.\end{cases},\ f(v_1v_2)=0 \text{ and } f(v_{4t+1}v_2)=1.$$

So we obtain $sum(0)=sum(1)=sum(2)=5t+3$, $sum(3)=5t+2$. Thus $|sum(x)-sum(y)|\le 1$ for all $x\neq y\in\mathbb{Z}_4$. Consequently f is 4-TEPC labeling.

Case 3.2: When t is even, we define $f: E(\mathcal{G})\to\mathbb{Z}_4$ as

$$f(uv_x)=\begin{cases}0, & if \quad 1\le x\le \frac{3t}{2};\\ 2, & if \quad \frac{3t+2}{2}\le x\le 3t;\\ 1, & if \quad 3t+1\le x\le 4t+2.\end{cases}$$

$$f(v_xv_{x+1})=\begin{cases}1, & if \quad 2\le x\le t;\\ 3, & if \quad t+1\le x\le 6t+2;\\ 2, & if \quad 6t+3\le x\le 8t+3.\end{cases},\ f(v_1v_2)=0 \text{ and } f(v_{4t+2}v_1)=0.$$

So we obtain $sum(3)=5t+2$, $sum(0)=sum(1)=sum(2)=5t+3$. Thus $|sum(x)-sum(y)|\le 1$ for all $x\neq y\in\mathbb{Z}_4$. Consequently f is 4-TEPC labeling.

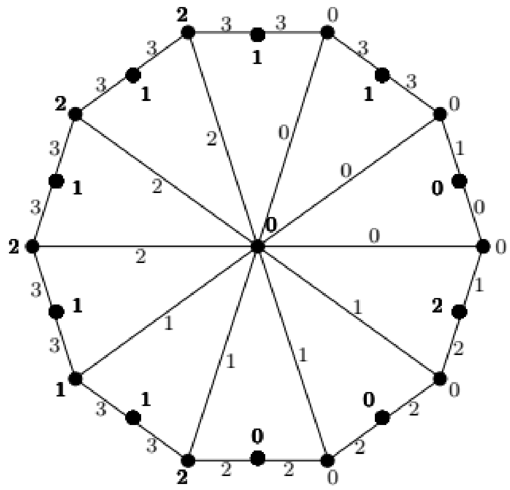

FIGURE 10. 4-TEPC labeling of $\mathcal{G}_{10}$

Case 4: Let $n \equiv 3 \pmod 4$ which implies $n = 4t+3$, for any integer $t \geq 1$.

Case 4.1: When t is odd, we define $f: E(\mathcal{G}) \to \mathbb{Z}_4$ as:

$$f(uv_x) = \begin{cases} 0, & if \quad 1 \leq x \leq \frac{3t-1}{2}; \\ 2, & if \quad \frac{3t+1}{2} \leq x \leq 3t-1; \\ 1, & if \quad 3t \leq x \leq 4t+3. \end{cases}$$

$$f(v_x v_{x+1}) = \begin{cases} 1, & if \quad 3 \leq x \leq t; \\ 3, & if \quad t+1 \leq x \leq 6t+3; \\ 2, & if \quad 6t+4 \leq x \leq 8t+6. \end{cases}, f(v_1v_2) = 0, \ f(v_2v_3) = 2 \text{ and } f(v_{4t+2}v_1) = 0.$$

So we obtain $sum(0) = sum(1) = sum(2) = sum(3) = 5t+4$. Thus $|sum(x) - sum(y)| \leq 1$ for all $x \neq y \in \mathbb{Z}_4$. Consequently f is 4-TEPC labeling.

Case 4.2: When t is even, we define $f: E(\mathcal{G}) \to \mathbb{Z}_4$ as

$$f(uv_x) = \begin{cases} 0, & if \quad 1 \leq x \leq \frac{3t}{2}; \\ 2, & if \quad \frac{3t+2}{2} \leq x \leq 4t+2. \end{cases}, \ f(uv_{4t+3}) = 1.$$

$$f(v_x v_{x+1}) = \begin{cases} 0, & if \quad 1 \leq x \leq t+2; \\ 3, & if \quad t+3 \leq x \leq 6t+6; \\ 1, & if \quad 6t+7 \leq x \leq 8t+5. \end{cases}, \ f(v_{4t+3}v_1) = 0.$$

So we obtain $sum(0) = sum(1) = sum(2) = sum(3) = 5t+4$. Thus $|sum(x) - sum(y)| \leq 1$ for all $x \neq y \in \mathbb{Z}_4$. Consequently f is 4-TEPC labeling.

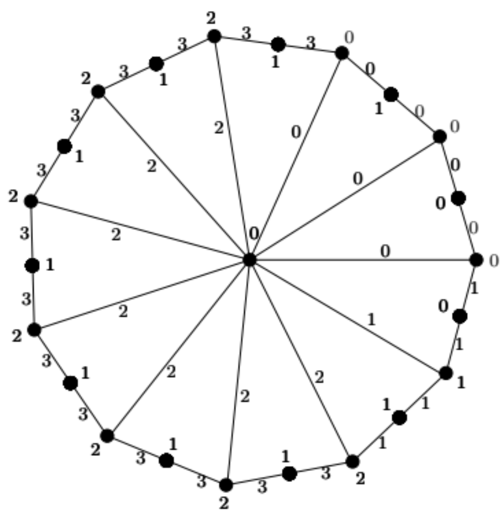

FIGURE 11. 4-TEPC labeling of $\mathcal{G}_{11}$

2.3. 4-TEPC labeling of Helm graph

Theorem 2.3. Let $\mathcal{G}$ be a helm graph H_n of $n+1$ vertices then $\mathcal{G}$ admits 4-TEPC labeling.

Proof. Let $V(\mathcal{G}) = \{u, v_x, w_n \ 1 \le x \le n\}$ and $E(\mathcal{G}) = \{uv_x,\ 1 \le x \le n\} \cup \{v_x v_{x+1},\ 1 \le x \le n-1\} \cup \{v_x w_x,\ 1 \le x \le n\} \cup \{v_n v_1\}$ be the vertex-set and edge-set of the wheel graph respectively as shown in Figure 3. Now we have the following four cases:

Case 1: Let n ≡ 0 (mod 4) which implies $n = 4t$, for any integer t ≥ 1. We define $f: E(\mathcal{G}) \to \mathbb{Z}_4$ as

$$f(uv_x) = \begin{cases} 0, & if \quad 1 \le x \le t; \\ 2, & if \quad t+1 \le x \le 3t; \\ 1, & if \quad 3t+1 \le x \le 4t. \end{cases}, \quad f(v_x w_x) = \begin{cases} 3, & if \quad 1 \le x \le t; \\ 1, & if \quad t+1 \le x \le 3t; \\ 0, & if \quad 3t+1 \le x \le 4t. \end{cases}$$

$$f(v_x v_{x+1}) = \begin{cases} 3, & if \quad 1 \le x \le 3t; \\ 2, & if \quad 3t+1 \le x \le 4t-1. \end{cases} \quad \text{and} \quad f(v_{4t} v_1) = 2.$$

So we obtain $sum(0) = 5t+1,\ sum(1) = sum(2) = sum(3) = 5t$. Thus $|sum(x) - sum(y)| \le 1$ for all $x \ne y \in \mathbb{Z}_4$. Consequently f is 4-TEPC labeling.

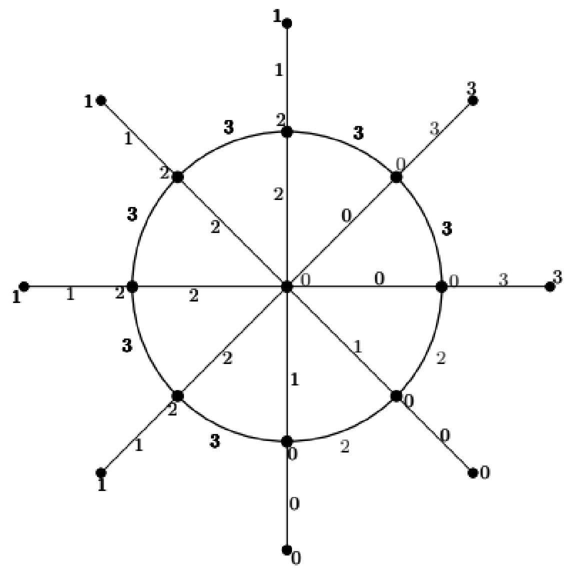

FIGURE 12. 4-TEPC labeling of H_8

Case 2: Let $n \equiv 1$ (mod 4) which implies $n = 4t+1$, for any integer t ≥ 1. *We* define $f: E(\mathcal{G}) \to \mathbb{Z}_4$ as

$$f(uv_x) = \begin{cases} 0, & if \quad 1 \le x \le t; \\ 2, & if \quad t+1 \le x \le 3t+1; \\ 1, & if \quad 3t+2 \le x \le 4t+1. \end{cases}, \quad f(v_x w_x) = \begin{cases} 3, & if \quad 1 \le x \le t; \\ 1, & if \quad t+1 \le x \le 3t+1; \\ 0, & if \quad 3t+2 \le x \le 4t+1. \end{cases}$$

$$f(v_x v_{x+1}) = \begin{cases} 3, & if \quad 1 \le x \le 3t+1; \\ 2, & if \quad 3t+2 \le x \le 4t. \end{cases} \quad \text{and} \quad f(v_{4t+1} v_1) = 2.$$

So we obtain $sum(0) = sum(3) = 5t+1,\ sum(1) = sum(2) = 5t+2$. Thus $|sum(x) - sum(y)| \le 1$ for all $x \ne y \in \mathbb{Z}_4$. Consequently f is 4-TEPC labeling.

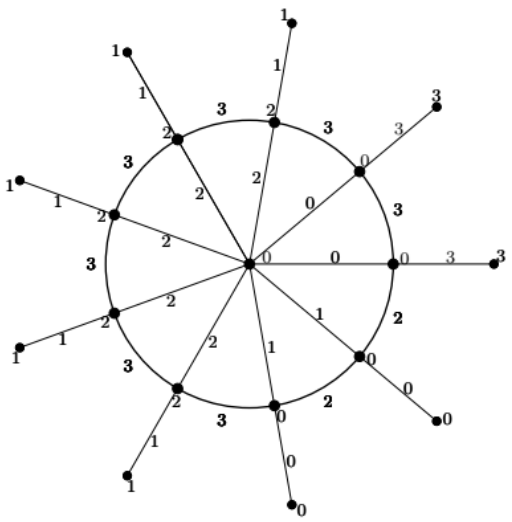

FIGURE 13. 4-TEPC labeling of H_9

Case 3: Let $n \equiv 2 \pmod 4$ which implies $n = 4t + 2$, for any integer $t \geq 1$. *We* define $f: E(\mathcal{G}) \to \mathbb{Z}_4$ as

$$f(uv_x) = \begin{cases} 0, & if \quad 1 \leq x \leq t+1; \\ 2, & if \quad t+2 \leq x \leq 3t+2; \\ 1, & if \quad 3t+3 \leq x \leq 4t+2. \end{cases}, \quad f(v_x w_x) = \begin{cases} 3, & if \quad 1 \leq x \leq t+1; \\ 1, & if \quad t+2 \leq x \leq 3t+2; \\ 0, & if \quad 3t+3 \leq x \leq 4t+2. \end{cases}$$

$$f(v_x v_{x+1}) = \begin{cases} 2, & if \quad 1 \leq x \leq t; \\ 3, & if \quad t+1 \leq x \leq 4t+1. \end{cases} \quad \text{and} \quad f(v_{4t+2} v_1) = 2.$$

So we obtain $sum(0) = sum(2) = sum(3) = 5t + 3, sum(1) = 5t + 2$. Thus $|sum(x) - sum(y)| \leq 1$ for all $x \neq y \in \mathbb{Z}_4$. Consequently f is 4-TEPC labeling.

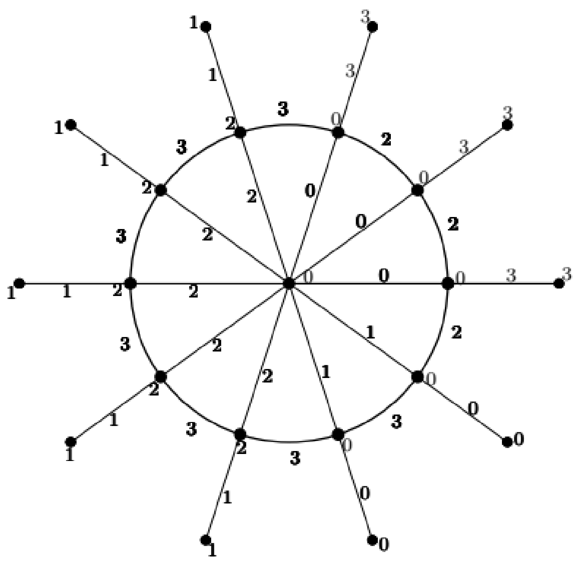

FIGURE 14. 4-TEPC labeling of H_{10}

Case 4: Let $n \equiv 3 \pmod 4$ which implies $n = 4t + 3$, for any integer $t \geq 1$. *We* define $f: E(\mathcal{G}) \to \mathbb{Z}_4$ as

$$f(uv_x) = \begin{cases} 0, & if \quad 1 \leq x \leq t+1; \\ 2, & if \quad t+2 \leq x \leq 3t+3; \\ 1, & if \quad 3t+4 \leq x \leq 4t+3. \end{cases}, \quad f(v_x w_x) = \begin{cases} 3, & if \quad 1 \leq x \leq t+1; \\ 1, & if \quad t+2 \leq x \leq 3t+3; \\ 0, & if \quad 3t+4 \leq x \leq 4t+3. \end{cases}$$

$$f(v_x v_{x+1}) = \begin{cases} 2, & if \quad 1 \leq x \leq t; \\ 3, & if \quad t+1 \leq x \leq 4t+2. \end{cases} \quad \text{and} \quad f(v_{4t+3} v_1) = 0.$$

So we obtain $sum(0) = sum(1) = sum(2) = sum(3) = 5t + 4$. Thus $|sum(x) - sum(y)| \leq 1$ for all $x \neq y \in \mathbb{Z}_4$. Consequently f is 4-TEPC labeling.

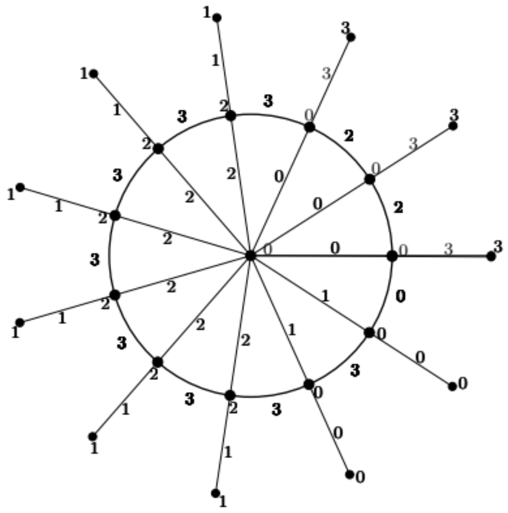

FIGURE 15. 4-TEPC labeling of H_{11}

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