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Research Article

On the Connections between Certain Class of New Quadruple and Known Triple Hyper-geometric Series

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Abstract: We aim in this work at establishing interesting operational connections between new quadruple hypergeometric series $X_i^{(4)}(i=1,\ldots,30)$ defined in [9] and certain class of triple series involving of Exton's functions X_1 to X_{20}, Srivastava's functions H_A, H_B, H_C, Lauricella's functions $F_A^{(3)}, F_B^{(3)}$, F_E, F_F, F_P and the general triple hypergeometric series $F^{(3)}[x,y,z]$. Some particular cases and consequences of our main results are also considered.

***Keywords**:* Operational connections; Exton's triple hypergeometric series; Lauricella's triple hypergeometric functions; Srivastava's triple hypergeometric functions; Quadruple hypergeometric series.

1.INTRODUCTION

In 1982 Exton [8] published a very interesting and useful research paper in which he encountered a number of triple hypergeometric functions of second order whose series representations involve such products as $(a)_{2m+2n+p}$ and $(a)_{2m+n+p}$, and introduced a set of 20 distinct triple hypergeometric functions X_1 to X_{20} and also gave their integral representations of Laplacian type which include the confluent hypergeometric functions ${}_1F_1$, ${}_0F_1$, a Humbert function Φ_2 and a Humbert function Ψ_2 in their kernels. It is not out of place to mention here that Exton's functions X_1 to X_{20} have been studied a lot until today; see, for example, the works [10, 3-6, 11–13]. Moreover, Exton [8] presented a large number of very interesting transformation formulas and reducible cases with the help of two known results which are called in the literature Kummer's first and second transformations or theorems.

Motivated essentially by the works by Exton ([7, Chapter 3], [8]) and Sharma and Parihar [15], we introduced in [9] thirty new and interesting quadruple hypergeometric series $X_i^{(4)}(i=1,\ldots,30)$ as follows:

$$X_1^{(4)}(a_1,a_1,a_1,a_1,a_1,a_1,a_2,a_2;c_2,c_1,c_1,c_3;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+2n+p+q}(a_2)_{p+q}}{(c_1)_{n+p}(c_2)_m(c_3)_q}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.1}$$

$$X_2^{(4)}(a_1,a_1,a_2,a_1,a_1,a_2,a_2,a_2;c_1,c_2,c_3,c_4;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+n+q}(a_2)_{q+n+2p}}{(c_1)_m(c_2)_n(c_3)_p(c_4)_q}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.2}$$

$$X_3^{(4)}(a_1,a_1,a_2,a_1,a_1,a_2,a_2,a_2;c_1,c_1,c_2,c_3;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+n+q}(a_2)_{q+n+2p}}{(c_1)_{m+n}(c_2)_p(c_3)_q}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.3}$$

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$$X_4^{(4)}(a_1,a_1,a_2,a_1,a_1,a_2,a_2,a_2;c_1,c_2,c_1,c_3;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+n+q}(a_2)_{q+n+2p}}{(c_1)_{m+p}(c_2)_n(c_3)_q}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.4}$$

$$X_5^{(4)}(a_1,a_1,a_2,a_1,a_1,a_2,a_2,a_2;c_1,c_1,c_1,c_2;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+n+q}(a_2)_{q+n+2p}}{(c_1)_{m+n+p}(c_2)_q}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.5}$$

$$X_6^{(4)}(a_1,a_1,a_2,a_1,a_1,a_2,a_2,a_2;c_1,c_1,c_2,c_2;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+n+q}(a_2)_{q+n+2p}}{(c_1)_{m+n}(c_2)_{p+q}}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.6}$$

$$X_7^{(4)}(a_1,a_1,a_1,a_1,a_1,a_2,a_3,a_1;c_1,c_2,c_3,c_4;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+2q+n+p}(a_2)_n(a_3)_p}{(c_1)_m(c_2)_n(c_3)_p(c_4)_q}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.7}$$

$$X_8^{(4)}(a_1,a_1,a_1,a_1,a_1,a_2,a_3,a_1;c_1,c_1,c_2,c_3;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+2q+n+p}(a_2)_n(a_3)_p}{(c_1)_{m+n}(c_2)_p(c_3)_q}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.8}$$

$$X_9^{(4)}(a_1,a_1,a_1,a_1,a_1,a_2,a_3,a_1;c_2,c_1,c_1,c_3;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+2q+n+p}(a_2)_n(a_3)_p}{(c_1)_{n+p}(c_2)_m(c_3)_q}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.9}$$

$$X_{10}^{(4)}(a_1,a_1,a_1,a_2,a_1,a_2,a_3,a_3;c_1,c_2,c_3,c_4;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+n+p}(a_2)_{n+q}(a_3)_{p+q}}{(c_1)_m(c_2)_n(c_3)_p(c_4)_q}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.10}$$

$$X_{11}^{(4)}(a_1,a_1,a_1,a_2,a_1,a_2,a_3,a_3;c_1,c_1,c_2,c_3;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+n+p}(a_2)_{n+q}(a_3)_{p+q}}{(c_1)_{m+n}(c_2)_p(c_3)_q}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.11}$$

$$X_{12}^{(4)}(a_1,a_1,a_1,a_2,a_1,a_2,a_3,a_3;c_2,c_1,c_1,c_3;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+n+p}(a_2)_{n+q}(a_3)_{p+q}}{(c_1)_{n+p}(c_2)_m(c_3)_q}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.12}$$

$$X_{13}^{(4)}(a_1,a_1,a_1,a_2,a_1,a_2,a_3,a_3;c_1,c_1,c_2,c_2;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+n+p}(a_2)_{n+q}(a_3)_{p+q}}{(c_1)_{m+n}(c_2)_{p+q}}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.13}$$

$$X_{14}^{(4)}(a_1,a_1,a_1,a_2,a_1,a_2,a_3,a_3;c_1,c_1,c_1,c_2;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+n+p}(a_2)_{n+q}(a_3)_{p+q}}{(c_1)_{m+n+p}(c_2)_q}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.14}$$

$$X_{15}^{(4)}(a_1,a_1,a_1,a_2,a_1,a_2,a_3,a_3;c,c,c,c;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+n+p}(a_2)_{n+q}(a_3)_{p+q}}{(c_1)_{m+n+p+q}}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.15}$$

$$X_{16}^{(4)}(a_1,a_1,a_1,a_1,a_1,a_2,a_3,a_2;c_1,c_1,c_2,c_2;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+n+p+q}(a_2)_{q+n}(a_3)_p}{(c_1)_{m+n}(c_2)_p(c_3)_q}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.16}$$

$$X_{17}^{(4)}(a_1,a_1,a_1,a_1,a_1,a_2,a_3,a_2;c_2,c_1,c_1,c_3;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+n+p+q}(a_2)_{q+n}(a_3)_p}{(c_1)_{n+p}(c_2)_m(c_3)_q}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.17}$$

$$X_{18}^{(4)}(a_1,a_1,a_1,a_1,a_1,a_2,a_3,a_2;c_1,c_1,c_2,c_2;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+n+p+q}(a_2)_{q+n}(a_3)_p}{(c_1)_{m+n}(c_2)_{p+q}}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.18}$$

$$X_{19}^{(4)}(a_1,a_1,a_1,a_1,a_1,a_2,a_3,a_2;c_1,c_1,c_1,c_2;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+n+p+q}(a_2)_{q+n}(a_3)_p}{(c_1)_{m+n+p}(c_2)_q}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.19}$$

$$X_{20}^{(4)}(a_1,a_1,a_2,a_1,a_1,a_2,a_3,a_2;c_1,c_2,c_3,c_4;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+n+q}(a_2)_{q+n+p}(a_3)_p}{(c_1)_m(c_2)_n(c_3)_p(c_4)_q}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.20}$$

$$X_{21}^{(4)}(a_1,a_1,a_2,a_1,a_1,a_2,a_3,a_2;c_1,c_1,c_2,c_3;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+n+q}(a_2)_{q+n+p}(a_3)_p}{(c_1)_{m+n}(c_2)_p(c_3)_q}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.21}$$

$$X_{22}^{(4)}(a_1,a_1,a_2,a_1,a_1,a_2,a_3,a_2;c_2,c_1,c_1,c_3;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+n+q}(a_2)_{q+n+p}(a_3)_p}{(c_1)_{n+p}(c_2)_m(c_3)_q}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.22}$$

$$X_{23}^{(4)}(a_1,a_1,a_2,a_1,a_1,a_2,a_3,a_2;c_1,c_1,c_1,c_2;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+n+q}(a_2)_{q+n+p}(a_3)_p}{(c_1)_{m+n+p}(c_2)_q}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.23}$$

$$X_{24}^{(4)}(a_1,a_1,a_2,a_1,a_1,a_2,a_3,a_3;c_1,c_2,c_1,c_3;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+n+q}(a_2)_{n+p}(a_3)_{p+q}}{(c_1)_{m+p}(c_2)_n(a_3)_q}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.24}$$

$$X_{25}^{(4)}(a_1,a_1,a_3,a_1,a_1,a_2,a_4,a_2;c_1,c_2,c_1,c_3;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+n+q}(a_2)_{n+q}(a_3)_p(a_4)_p}{(c_1)_{m+p}(c_2)_n(c_3)_q}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.25}$$

$$X_{26}^{(4)}(a_1,a_1,a_3,a_1,a_1,a_2,a_4,a_4;c_1,c_1,c_1,c_2;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+n+q}(a_2)_n(a_3)_p(a_4)_{p+q}}{(c_1)_{m+n+p}(c_2)_q}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.26}$$

$$X_{27}^{(4)}(a_1,a_1,a_3,a_1,a_1,a_2,a_4,a_4;c_2,c_1,c_1,c_3;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+n+q}(a_2)_n(a_3)_p(a_4)_{p+q}}{(c_1)_{n+p}(c_2)_m(c_3)_q}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.27}$$

$$X_{28}^{(4)}(a_1,a_1,a_1,a_1,a_1,a_2,a_3,a_4;c_1,c_1,c_2,c_3;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+n+p+q}(a_2)_n(a_3)_p(a_4)_q}{(c_1)_{m+n}(c_2)_p(c_3)_q}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.28}$$

$$X_{29}^{(4)}(a_1,a_1,a_3,a_4,a_1,a_2,a_5,a_6;c_1,c_2,c_1,c_1;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+n}(a_2)_n(a_3)_p(a_4)_q(a_5)_p(a_6)_q}{(c_1)_{m+p+q}(c_2)_n}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}; \tag{1.29}$$

$$X_{30}^{(4)}(a_1,a_1,a_3,a_4,a_1,a_2,a_5,a_6;c,c,c,c;x,y,z,u)=\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+n}(a_2)_n(a_3)_p(a_4)_q(a_5)_p(a_6)_q}{(c)_{m+n+p+q}}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}\frac{u^q}{q!}. \tag{1.30}$$

Here, in this paper, we aim at establishing interesting operational connections between quadruple hypergeometric series $X_i^{(4)}(i=1,\ldots,30)$ and the triple series of Exton's functions X_1 to X_{20}, Srivastava's functions H_A, H_B, H_C (see, for details [16] and [17]), Lauricella's functions $F_A^{(3)}, F_B^{(3)}, F_E, F_F, F_P$ (cf. [16, Sections 1.4 and 1.5] and [17, Chapter 1]) and the general triple hypergeometric series $F^{(3)}[x,y,z]$ (see e.g. [16, p. 44, (14) and (15)]. Our results presented here are derived with the help of two formulas:

$$D_x^m x^\lambda=\frac{\Gamma(\lambda+1)}{\Gamma(\lambda-m+1)}x^{\lambda-m}, \tag{1.31}$$

$$D_x^{-m} x^\lambda=\frac{\Gamma(\lambda+1)}{\Gamma(\lambda+m+1)}x^{\lambda+m}, \tag{1.32}$$

$m\in N\cup\{0\},\ \lambda\in C-\{-1,-2,...\}$, where the operators D_x and D_x^{-1} denote the derivative operator and the inverse of the derivative, respectively (see, for example, [14],[1] and [2]).

2. Main formulae

For convenience and simplicity, let $\sigma=(1-u\,[D_\alpha\,\beta^{-1}D_\beta^{-1}\alpha])$, then the results to be established here are as follows :

$$\left(1-u\left[D_\alpha\,\beta^{-1}D_\beta^{-1}\,\alpha\right]\right)^{-a_1} X_1\left(a_1,a_2;c_2,c_1;\frac{x}{\sigma^2},\frac{y}{\sigma^2},\frac{\alpha z}{\sigma}\right)\left\{\alpha^{a_2-1}\beta^{c_3-1}\right\} \tag{2.1}$$
$$=\alpha^{a_2-1}\beta^{c_3-1}\,X_1^{(4)}(a_1,a_1,a_1,a_1,a_1,a_1,a_2,a_2;c_2,c_1,c_1,c_3;x,y,\alpha z,u);$$

$$\left(1-u\left[D_\alpha\,\beta^{-1}D_\beta^{-1}\,\alpha\right]\right)^{-a_1} X_{12}\left(a_1,a_2;c_1,c_2,c_3;\frac{x}{\sigma^2},\frac{\alpha y}{\sigma},\alpha^2 z\right)\left\{\alpha^{a_2-1}\beta^{c_4-1}\right\} \tag{2.2}$$
$$=\alpha^{a_2-1}\beta^{c_4-1}\,X_2^{(4)}(a_1,a_1,a_2,a_1,a_1,a_2,a_2,a_2;c_1,c_2,c_3,c_4;x,\alpha y,\alpha^2 z,u);$$

$$\left(1-u\left[D_\alpha\,\beta^{-1}D_\beta^{-1}\,\alpha\right]\right)^{-a_1} X_{10}\left(a_1,a_2;c_1,c_2;\frac{x}{\sigma^2},\frac{\alpha y}{\sigma},\alpha^2 z\right)\left\{\alpha^{a_2-1}\beta^{c_3-1}\right\} \tag{2.3}$$
$$=\alpha^{a_2-1}\beta^{c_3-1}\,X_3^{(4)}(a_1,a_1,a_2,a_1,a_1,a_2,a_2,a_2;c_1,c_1,c_2,c_3;x,\alpha y,\alpha^2 z,u);$$

$$\left(1-u\left[D_\alpha\,\beta^{-1}D_\beta^{-1}\,\alpha\right]\right)^{-a_1} X_{11}\left(a_1,a_2;c_1,c_2;\frac{x}{\sigma^2},\frac{\alpha y}{\sigma},\alpha^2 z\right)\left\{\alpha^{a_2-1}\beta^{c_3-1}\right\} \tag{2.4}$$
$$=\alpha^{a_2-1}\beta^{c_3-1}\,X_4^{(4)}(a_1,a_1,a_2,a_1,a_1,a_2,a_2,a_2;c_1,c_2,c_1,c_3;x,\alpha y,\alpha^2 z,u);$$

$$\left(1-u\left[D_\alpha\,\beta^{-1}D_\beta^{-1}\,\alpha\right]\right)^{-a_1} X_9\left(a_1,a_2;c_1;\frac{x}{\sigma^2},\frac{\alpha y}{\sigma},\alpha^2 z\right)\left\{\alpha^{a_2-1}\beta^{c_2-1}\right\} \tag{2.5}$$
$$=\alpha^{a_2-1}\beta^{c_2-1}\,X_5^{(4)}(a_1,a_1,a_2,a_1,a_1,a_2,a_2,a_2;c_1,c_1,c_1,c_2;x,\alpha y,\alpha^2 z,u);$$

$$\left(1-u\left[D_\alpha\,\beta^{-1}D_\beta^{-1}\,\alpha\right]\right)^{-a_2} X_{10}\left(a_1,a_2;c_1,c_2;\alpha^2 x,\frac{\alpha y}{\sigma},\frac{\beta z}{\sigma^2}\right)\left\{\alpha^{a_1-1}\beta^{c_2-1}\right\} \tag{2.6}$$
$$=\alpha^{a_1-1}\beta^{c_2-1}\,X_6^{(4)}(a_1,a_1,a_2,a_1,a_1,a_2,a_2,a_2;c_1,c_1,c_2,c_2;\alpha^2 x,\alpha y,\beta z,u);$$

$$\left(1-u\left[D_\alpha\,\beta^{-1}D_\beta^{-1}\,\alpha\right]\right)^{-a_1} X_2\left(a_1,a_2;c_1,c_2,c_3;\frac{x}{\sigma^2},\frac{y}{\sigma^2},\frac{z}{\sigma}\right)\left\{\alpha^{a_3-1}\beta^{c_4-1}\right\} \tag{2.7}$$
$$=\alpha^{a_3-1}\beta^{c_4-1}\,X_7^{(4)}(a_1,a_1,a_1,a_1,a_1,a_3,a_2,a_1;c_1,c_2,c_3,c_4;x,u,z,y);$$

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_1}\ X_2\left(a_1,a_2;c_1,c_2,c_3;\frac{\beta\,x}{\sigma^2},\frac{y}{\sigma^2},\frac{z}{\sigma}\right)\left\{\alpha^{a_3-1}\beta^{c_1-1}\right\}$$
$$=\alpha^{a_3-1}\beta^{c_1-1}\ X_8^{(4)}\left(a_1,a_1,a_1,a_1,a_1,a_3,a_2,a_1;c_1,c_1,c_3,c_2;\beta\,x,u,z,y\right); \tag{2.8}$$

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_1}\ X_2\left(a_1,a_2;c_1,c_2,c_3;\frac{x}{\sigma^2},\frac{y}{\sigma^2},\frac{\beta\,z}{\sigma}\right)\left\{\alpha^{a_3-1}\beta^{c_3-1}\right\}$$
$$=\alpha^{a_3-1}\beta^{c_3-1}\ X_9^{(4)}\left(a_1,a_1,a_1,a_1,a_1,a_3,a_2,a_1;c_1,c_3,c_3,c_2;x,u,\beta\,z,y\right); \tag{2.9}$$

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_2}\ X_8\left(a_1,a_2,a_3;c_1,c_2,c_3;x,\frac{y}{\sigma},\alpha\,z\right)\left\{\alpha^{a_3-1}\beta^{c_4-1}\right\}$$
$$=\alpha^{a_3-1}\beta^{c_4-1}\ X_{10}^{(4)}\left(a_1,a_1,a_1,a_2,a_1,a_2,a_3,a_3;c_1,c_2,c_3,c_4;x,y,\alpha\,z,u\right); \tag{2.10}$$

$$\left(1-u\left[D_{\alpha}^2\ \beta^{-1}D_{\beta}^{-1}\ \gamma^{-1}D_{\gamma}^{-1}\alpha^2\right]\right)^{-a}\ H_B\left(a_1,a_2,a_3;c_1,c_2,c_3;\alpha\,x,y,\alpha\,z\right)\left\{\alpha^{a_1-1}\ \beta^{c_4-1}\ \gamma^{a-1}\right\}$$
$$=\alpha^{a_1-1}\ \beta^{c_4-1}\ \gamma^{a-1}\ X_{10}^{(4)}\left(a_1,a_1,a_1,a_2,a_1,a_2,a_3,a_3;c_4,c_1,c_3,c_2;u,\alpha\,x,\alpha\,z,y\right); \tag{2.11}$$

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_2}\ X_6\left(a_1,a_2,a_3;c_1,c_2;x,\frac{y}{\sigma},\alpha\,z\right)\left\{\alpha^{a_3-1}\beta^{c_3-1}\right\}$$
$$=\alpha^{a_3-1}\beta^{c_3-1}\ X_{11}^{(4)}\left(a_1,a_1,a_1,a_2,a_1,a_2,a_3,a_3;c_1,c_1,c_2,c_3;x,y,\alpha\,z,u\right); \tag{2.12}$$

$$\left(1-u\left[D_{\alpha}^2\ \beta^{-1}D_{\beta}^{-1}\ \gamma^{-1}D_{\gamma}^{-1}\alpha^2\right]\right)^{-a}\ H_B\left(a_1,a_2,a_3;c_1,c_2,c_3;\alpha\beta\,x,y,\alpha\,z\right)\left\{\alpha^{a_1-1}\ \beta^{c_1-1}\ \gamma^{a-1}\right\}$$
$$=\alpha^{a_1-1}\ \beta^{c_1-1}\ \gamma^{a-1}\ X_{11}^{(4)}\left(a_1,a_1,a_1,a_2,a_1,a_2,a_3,a_3;c_1,c_1,c_3,c_2;u,\alpha\beta\,x,\alpha\,z,y\right); \tag{2.13}$$

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_2}\ X_7\left(a_1,a_2,a_3;c_2,c_1;x,\frac{y}{\sigma},\alpha\,z\right)\left\{\alpha^{a_3-1}\beta^{c_3-1}\right\}$$
$$=\alpha^{a_3-1}\beta^{c_3-1}\ X_{12}^{(4)}\left(a_1,a_1,a_1,a_2,a_1,a_2,a_3,a_3;c_2,c_1,c_1,c_3;x,y,\alpha\,z,u\right); \tag{2.14}$$

$$\left(1-u\left[D_{\alpha}^2\ \beta^{-1}D_{\beta}^{-1}\ \gamma^{-1}D_{\gamma}^{-1}\alpha^2\right]\right)^{-a}\ H_A\left(a_1,a_2,a_3;c_1,c_2;x,\alpha\,y,\alpha\,z\right)\left\{\alpha^{a_3-1}\ \beta^{c_3-1}\ \gamma^{a-1}\right\}$$
$$=\alpha^{a_3-1}\ \beta^{c_3-1}\ \gamma^{a-1}\ X_{12}^{(4)}\left(a_3,a_3,a_3,a_2,a_3,a_2,a_1,a_1;c_3,c_2,c_2,c_1;u,\alpha\,y,\alpha\,z,x\right); \tag{2.15}$$

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_3}\ X_6\left(a_1,a_2,a_3;c_1,c_2;x,\alpha\,y,\frac{\beta\,z}{\sigma}\right)\left\{\alpha^{a_2-1}\beta^{c_2-1}\right\}$$
$$=\alpha^{a_2-1}\beta^{c_2-1}\ X_{13}^{(4)}\left(a_1,a_1,a_1,a_2,a_1,a_2,a_3,a_3;c_1,c_1,c_2,c_2;x,\alpha\,y,\beta\,z,u\right); \tag{2.16}$$

$$\left(1-u\left[D_{\alpha}^2\ \beta^{-1}D_{\beta}^{-1}\ \gamma^{-1}D_{\gamma}^{-1}\alpha^2\right]\right)^{-a}\ H_A\left(a_1,a_2,a_3;c_1,c_2;\alpha\beta\,x,y,\alpha\,z\right)\left\{\alpha^{a_1-1}\ \beta^{c_1-1}\ \gamma^{a-1}\right\}$$
$$=\alpha^{a_1-1}\ \beta^{c_1-1}\ \gamma^{a-1}\ X_{13}^{(4)}\left(a_1,a_1,a_1,a_2,a_1,a_2,a_3,a_3;c_1,c_1,c_2,c_2;u,\alpha\beta\,x,\alpha\,z,y\right); \tag{2.17}$$

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_3}\ X_5\left(a_1,a_2,a_3;c_1;x,\alpha\,y,\frac{z}{\sigma}\right)\left\{\alpha^{a_2-1}\beta^{c_2-1}\right\}$$
$$=\alpha^{a_2-1}\beta^{c_2-1}\ X_{14}^{(4)}\left(a_1,a_1,a_1,a_2,a_1,a_2,a_3,a_3;c_1,c_1,c_1,c_2;x,\alpha\,y,z,u\right); \tag{2.18}$$

$$\left(1-u\left[D_{\alpha}^2\ \beta^{-1}D_{\beta}^{-1}\ \gamma^{-1}D_{\gamma}^{-1}\alpha^2\right]\right)^{-a}\ H_A\left(a_1,a_2,a_3;c_1,c_2;x,\alpha\beta\,y,\alpha\beta\,z\right)\left\{\alpha^{a_3-1}\ \beta^{c_2-1}\ \gamma^{a-1}\right\}$$
$$=\alpha^{a_3-1}\ \beta^{c_2-1}\ \gamma^{a-1}\ X_{14}^{(4)}\left(a_3,a_3,a_3,a_2,a_3,a_2,a_1,a_1;c_2,c_2,c_2,c_1;u,\alpha\beta\,y,\alpha\beta\,z,x\right); \tag{2.19}$$

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_2}\ X_5\left(a_1,a_2,a_3;c;\beta\,x,\frac{\beta\,y}{\sigma},\alpha\beta\,z\right)\left\{\alpha^{a_3-1}\beta^{c-1}\right\}$$
$$=\alpha^{a_3-1}\beta^{c-1}\ X_{15}^{(4)}\left(a_1,a_1,a_1,a_2,a_1,a_2,a_3,a_3;c,c,c,c;\beta\,x,\beta\,y,\alpha\beta\,z,u\right); \tag{2.20}$$

$$\left(1-u\left[D_{\alpha}^2\ \beta^{-1}D_{\beta}^{-1}\ \gamma^{-1}D_{\gamma}^{-1}\alpha^2\right]\right)^{-a}\ H_C\left(a_1,a_2,a_3;c;\alpha\beta\,x,\beta\,y,\alpha\beta\,z\right)\left\{\alpha^{a_1-1}\ \beta^{c-1}\ \gamma^{a-1}\right\}$$
$$=\alpha^{a_1-1}\ \beta^{c-1}\ \gamma^{a-1}\ X_{15}^{(4)}\left(a_1,a_1,a_1,a_2,a_1,a_2,a_3,a_3;c,c,c,c;u,\alpha\beta\,x,\alpha\beta\,z,\beta\,y\right); \tag{2.21}$$

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_1}\ X_3\left(a_1,a_2;c_1,c_2;\frac{x}{\sigma^2},\frac{y}{\sigma},\frac{z}{\sigma}\right)\left\{\alpha^{a_3-1}\beta^{c_3-1}\right\}$$
$$=\alpha^{a_3-1}\beta^{c_3-1}\ X_{16}^{(4)}\left(a_1,a_1,a_1,a_1,a_1,a_2,a_3,a_2;c_1,c_1,c_3,c_2;x,y,u,z\right); \quad (2.22)$$

$$\left(1-u\left[D_{\alpha}^2\ \beta^{-1}D_{\beta}^{-1}\ \gamma^{-1}D_{\gamma}^{-1}\alpha^2\right]\right)^{-a}\ F_E\left(a_1,a_1,a_1,a_2,a_3,a_3;c_1,c_2,c_3;\alpha\ x,\alpha\beta\ y,\alpha\ z\right)$$
$$\times\left\{\alpha^{a_1-1}\ \beta^{c_2-1}\ \gamma^{a-1}\right\} \quad (2.23)$$
$$=\alpha^{a_1-1}\ \beta^{c_2-1}\ \gamma^{a-1}\ X_{16}^{(4)}\left(a_1,a_1,a_1,a_1,a_1,a_3,a_2,a_3;c_2,c_2,c_1,c_3;u,\alpha\beta\ y,\alpha\ x,\alpha\ z\right);$$

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_1}\ X_4\left(a_1,a_2;c_1,c_2,c_3;\frac{x}{\sigma^2},\frac{\beta\ y}{\sigma},\frac{z}{\sigma}\right)\left\{\alpha^{a_3-1}\beta^{c_2-1}\right\}$$
$$=\alpha^{a_3-1}\beta^{c_2-1}\ X_{17}^{(4)}\left(a_1,a_1,a_1,a_1,a_1,a_2,a_3,a_2;c_1,c_2,c_2,c_3;x,\beta\ y,u,z\right); \quad (2.24)$$

$$\left(1-u\left[D_{\alpha}^2\ \beta^{-1}D_{\beta}^{-1}\ \gamma^{-1}D_{\gamma}^{-1}\alpha^2\right]\right)^{-a}\ F_F\left(a_1,a_1,a_1,a_2,a_3,a_2;c_1,c_2,c_2;\alpha\ x,\alpha\ y,\alpha\ z\right)$$
$$\times\left\{\alpha^{a_1-1}\ \beta^{c_3-1}\ \gamma^{a-1}\right\} \quad (2.25)$$
$$=\alpha^{a_1-1}\ \beta^{c_3-1}\ \gamma^{a-1}\ X_{17}^{(4)}\left(a_1,a_1,a_1,a_1,a_1,a_2,a_3,a_2;c_3,c_2,c_2,c_1;u,\alpha\ z,\alpha\ y,\alpha\ x\right);$$

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_1}\ X_7\left(a_1,a_2,a_3;c_2,c_1;\frac{\beta\ x}{\sigma^2},\frac{\alpha\ y}{\sigma},\frac{z}{\sigma}\right)\left\{\alpha^{a_2-1}\beta^{c_2-1}\right\}$$
$$=\alpha^{a_2-1}\beta^{c_2-1}\ X_{18}^{(4)}\left(a_1,a_1,a_1,a_1,a_1,a_2,a_3,a_2;c_2,c_2,c_1,c_1;\beta\ x,u,z,\alpha\ y\right); \quad (2.26)$$

$$\left(1-u\left[D_{\alpha}^2\ \beta^{-1}D_{\beta}^{-1}\ \gamma^{-1}D_{\gamma}^{-1}\alpha^2\right]\right)^{-a}\ F_F\left(a_1,a_1,a_1,a_2,a_3,a_2;c_1,c_2,c_2;\alpha\beta\ x,\alpha\ y,\alpha\ z\right)$$
$$\times\left\{\alpha^{a_1-1}\ \beta^{c_1-1}\ \gamma^{a-1}\right\}=\alpha^{a_1-1}\ \beta^{c_1-1}\ \gamma^{a-1}\ X_{18}^{(4)}\left(a_1,a_1,a_1,a_1,a_1,a_2,a_3,a_2;c_1,c_1,c_2,c_2;u,\alpha\beta\ x,\alpha\ y,\alpha\ z\right); \quad (2.27)$$

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_1}\ X_3\left(a_1,a_2;c_1,c_2;\frac{\beta\ x}{\sigma^2},\frac{\beta\ y}{\sigma},\frac{z}{\sigma}\right)\left\{\alpha^{a_3-1}\beta^{c_1-1}\right\}$$
$$=\alpha^{a_3-1}\beta^{c_1-1}\ X_{19}^{(4)}\left(a_1,a_1,a_1,a_1,a_1,a_2,a_3,a_2;c_1,c_1,c_1,c_2;\beta\ x,\beta\ y,u,z\right); \quad (2.28)$$

$$\left(1-u\left[D_{\alpha}^2\ \beta^{-1}D_{\beta}^{-1}\ \gamma^{-1}D_{\gamma}^{-1}\alpha^2\right]\right)^{-a}\ F_F\left(a_1,a_1,a_1,a_2,a_3,a_2;c_1,c_2,c_2;\alpha\ x,\alpha\beta\ y,\alpha\beta\ z\right)$$
$$\times\left\{\alpha^{a_1-1}\ \beta^{c_2-1}\ \gamma^{a-1}\right\} \quad (2.29)$$
$$=\alpha^{a_1-1}\ \beta^{c_2-1}\ \gamma^{a-1}\ X_{19}^{(4)}\left(a_1,a_1,a_1,a_1,a_1,a_2,a_3,a_2;c_2,c_2,c_2,c_1;u,\alpha\beta\ z,\alpha\beta\ y,\alpha\ x\right);$$

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_1}\ X_{17}\left(a_1,a_2,a_3;c_1,c_2,c_3;\frac{x}{\sigma^2},\frac{\alpha\ y}{\sigma},\alpha\ z\right)\left\{\alpha^{a_2-1}\beta^{c_4-1}\right\}$$
$$=\alpha^{a_2-1}\beta^{c_4-1}\ X_{20}^{(4)}\left(a_1,a_1,a_2,a_1,a_1,a_2,a_3,a_2;c_1,c_2,c_3,c_4;x,\alpha\ y,\alpha\ z,u\right); \quad (2.30)$$

$$\left(1-u\left[D_{\alpha}^2\ \beta^{-1}D_{\beta}^{-1}\ \gamma^{-1}D_{\gamma}^{-1}\alpha^2\right]\right)^{-a}\ F_E\left(a_1,a_1,a_1,a_2,a_3,a_3;c_1,c_2,c_3;x,\alpha\ y,\alpha\ z\right)$$
$$\times\left\{\alpha^{a_3-1}\ \beta^{c_4-1}\ \gamma^{a-1}\right\}=\alpha^{a_3-1}\ \beta^{c_4-1}\ \gamma^{a-1}\ X_{20}^{(4)}\left(a_3,a_3,a_1,a_3,a_3,a_1,a_2,a_1;c_4,c_2,c_1,c_3;u,\alpha\ y,x,\alpha\ z\right); \quad (2.31)$$

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_2}\ X_{14}\left(a_1,a_2,a_3;c_1,c_2;\alpha^2x,\frac{\alpha\ y}{\sigma},\frac{z}{\sigma}\right)\left\{\alpha^{a_1-1}\beta^{c_3-1}\right\}$$
$$=\alpha^{a_1-1}\beta^{c_3-1}\ X_{21}^{(4)}\left(a_1,a_1,a_2,a_1,a_1,a_2,a_3,a_2;c_1,c_1,c_2,c_3;\alpha^2x,\alpha\ y,z,u\right); \quad (2.32)$$

$$\left(1-u\left[D_{\alpha}^2\ \beta^{-1}D_{\beta}^{-1}\ \gamma^{-1}D_{\gamma}^{-1}\alpha^2\right]\right)^{-a}\ F_E\left(a_1,a_1,a_1,a_2,a_3,a_3;c_1,c_2,c_3;x,\alpha\beta\ y,\alpha\ z\right)$$
$$\times\left\{\alpha^{a_3-1}\ \beta^{c_2-1}\ \gamma^{a-1}\right\}=\alpha^{a_3-1}\ \beta^{c_2-1}\ \gamma^{a-1}\ X_{21}^{(4)}\left(a_3,a_3,a_1,a_3,a_3,a_1,a_2,a_1;c_2,c_2,c_1,c_3;u,\alpha\beta\ y,x,\alpha\ z\right); \quad (2.33)$$

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_2}\ X_{15}\left(a_1,a_2,a_3;c_2,c_1;\alpha^2x,\frac{\alpha\ y}{\sigma},\frac{z}{\sigma}\right)\left\{\alpha^{a_1-1}\beta^{c_3-1}\right\}$$
$$=\alpha^{a_1-1}\beta^{c_3-1}\ X_{22}^{(4)}\left(a_1,a_1,a_2,a_1,a_1,a_2,a_3,a_2;c_2,c_1,c_1,c_3;\alpha^2x,\alpha\ y,z,u\right); \quad (2.34)$$

$$\left(1-u\left[D_{\alpha}^{2}\ \beta^{-1}D_{\beta}^{-1}\ \gamma^{-1}D_{\gamma}^{-1}\alpha^{2}\right]\right)^{-a}\ F_{F}\left(a_1,a_1,a_1,a_2,a_3,a_2;c_1,c_2,c_2;\alpha\ x,y,\alpha\ z\right)\left\{\alpha^{a_2-1}\ \beta^{c_3-1}\ \gamma^{a-1}\right\}$$
$$=\alpha^{a_2-1}\ \beta^{c_3-1}\ \gamma^{a-1}\ X_{22}^{(4)}\left(a_2,a_2,a_1,a_2,a_2,a_1,a_3,a_1;c_3,c_2,c_2,c_1;u,\alpha\ z,\ y,\alpha\ x\right); \quad (2.35)$$

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_2}\ X_{13}\left(a_1,a_2,a_3;c_1;\alpha^2 x,\frac{\alpha\ y}{\sigma},\frac{z}{\sigma}\right)\left\{\alpha^{a_1-1}\beta^{c_2-1}\right\}$$
$$=\alpha^{a_1-1}\beta^{c_2-1}\ X_{23}^{(4)}\left(a_1,a_1,a_2,a_1,a_1,a_2,a_3,a_2;c_1,c_1,c_1,c_2;\alpha^2 x,\alpha\ y,z,u\right); \quad (2.36)$$

$$\left(1-u\left[D_{\alpha}^{2}\ \beta^{-1}D_{\beta}^{-1}\ \gamma^{-1}D_{\gamma}^{-1}\alpha^{2}\right]\right)^{-a}\ F_{F}\left(a_1,a_1,a_1,a_2,a_3,a_2;c_1,c_2,c_2;\alpha\ x,\beta\ y,\alpha\beta\ z\right)\left\{\alpha^{a_2-1}\ \beta^{c_2-1}\ \gamma^{a-1}\right\}$$
$$=\alpha^{a_2-1}\ \beta^{c_2-1}\ \gamma^{a-1}\ X_{23}^{(4)}\left(a_2,a_2,a_1,a_2,a_2,a_1,a_3,a_1;c_2,c_2,c_2,c_1;u,\alpha\beta z,\beta y,\alpha x\right); \quad (2.37)$$

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_1}\ X_{16}\left(a_1,a_2,a_3;c_1,c_2;\frac{x}{\sigma^2},\frac{y}{\sigma},\alpha\ z\right)\left\{\alpha^{a_3-1}\beta^{c_3-1}\right\}$$
$$=\alpha^{a_3-1}\beta^{c_3-1}\ X_{24}^{(4)}\left(a_1,a_1,a_2,a_1,a_1,a_2,a_3,a_3;c_1,c_2,c_1,c_3;x,y,\alpha\ z,u\right); \quad (2.38)$$

$$\left(1-u\left[D_{\alpha}^{2}\ \beta^{-1}D_{\beta}^{-1}\ \gamma^{-1}D_{\gamma}^{-1}\alpha^{2}\right]\right)^{-a}\ H_{B}\left(a_1,a_2,a_3;c_1,c_2,c_3;\alpha\ x,\beta\ y,\alpha\ z\right)\left\{\alpha^{a_1-1}\ \beta^{c_2-1}\ \gamma^{a-1}\right\}$$
$$=\alpha^{a_1-1}\ \beta^{c_2-1}\ \gamma^{a-1}\ X_{24}^{(4)}\left(a_1,a_1,a_2,a_1,a_1,a_2,a_3,a_3;c_2,c_1,c_2,c_3;u,\alpha\ x,\beta\ y,\alpha\ z\right); \quad (2.39)$$

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_1}\ X_{20}\left(a_1,a_2,a_3,a_4;c_1,c_2;\frac{x}{\sigma^2},\frac{\alpha\ y}{\sigma},z\right)\left\{\alpha^{a_2-1}\beta^{c_3-1}\right\}$$
$$=\alpha^{a_2-1}\beta^{c_3-1}\ X_{25}^{(4)}\left(a_1,a_1,a_3,a_1,a_1,a_2,a_4,a_2;c_1,c_2,c_1,c_3;x,\alpha\ y,z,u\right); \quad (2.40)$$

$$\left(1-u\left[D_{\alpha}^{2}\ \beta^{-1}D_{\beta}^{-1}\ \gamma^{-1}D_{\gamma}^{-1}\alpha^{2}\right]\right)^{-a}\ F^{(3)}\left[\begin{matrix}-::a_1,a_2;-;-:-;-;a_3,a_4; & \\ & \alpha\ x,\alpha\ y,\beta\ z\\ -::\quad -;-;-:c_1;c_2;\quad c_3; & \end{matrix}\right]\left\{\alpha^{a_1-1}\ \beta^{c_3-1}\ \gamma^{a-1}\right\}$$
$$=\alpha^{a_1-1}\ \beta^{c_3-1}\ \gamma^{a-1}\ X_{25}^{(4)}\left(a_1,a_1,a_3,a_1,a_1,a_2,a_4,a_2;c_3,c_1,c_3,c_2;u,\alpha\ x,\beta\ z,\alpha\ y\right); \quad (2.41)$$

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_1}\ X_{18}\left(a_1,a_2,a_3,a_4;c_1;\frac{x}{\sigma^2},\frac{y}{\sigma},\alpha\ z\right)\left\{\alpha^{a_4-1}\beta^{c_2-1}\right\}$$
$$=\alpha^{a_4-1}\beta^{c_2-1}\ X_{26}^{(4)}\left(a_1,a_1,a_3,a_1,a_1,a_2,a_4,a_4;c_1,c_1,c_1,c_2;x,y,\alpha\ z,u\right); \quad (2.42)$$

$$\left(1-u\left[D_{\alpha}^{2}\ \beta^{-1}D_{\beta}^{-1}\ \gamma^{-1}D_{\gamma}^{-1}\alpha^{2}\right]\right)^{-a}\ F_{P}\left(a_1,a_2,a_1,a_3,a_3,a_4;c_1,c_2,c_2;\alpha\ x,\beta\ y,\alpha\beta\ z\right)\left\{\alpha^{a_1-1}\ \beta^{c_2-1}\ \gamma^{a-1}\right\}$$
$$=\alpha^{a_1-1}\ \beta^{c_2-1}\ \gamma^{a-1}\ X_{26}^{(4)}\left(a_1,a_1,a_2,a_1,a_1,a_4,a_3,a_3;c_2,c_2,c_2,c_1;u,\alpha\beta\ z,\beta\ y,\alpha\ x\right); \quad (2.43)$$

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_4}\ X_{19}\left(a_1,a_2,a_3,a_4;c_2,c_1;\alpha^2 x,\alpha\ y,\frac{z}{\sigma}\right)\left\{\alpha^{a_1-1}\beta^{c_3-1}\right\}$$
$$=\alpha^{a_1-1}\beta^{c_3-1}\ X_{27}^{(4)}\left(a_1,a_1,a_3,a_1,a_1,a_2,a_4,a_4;c_2,c_1,c_1,c_3;\alpha^2 x,\alpha\ y,z,u\right); \quad (2.44)$$

$$\left(1-u\left[D_{\alpha}^{2}\ \beta^{-1}D_{\beta}^{-1}\ \gamma^{-1}D_{\gamma}^{-1}\alpha^{2}\right]\right)^{-a}\ F_{P}\left(a_1,a_2,a_1,a_3,a_3,a_4;c_1,c_2,c_2;\alpha\ x,y,\alpha\ z\right)\left\{\alpha^{a_1-1}\ \beta^{c_3-1}\ \gamma^{a-1}\right\}$$
$$=\alpha^{a_1-1}\ \beta^{c_3-1}\ \gamma^{a-1}\ X_{27}^{(4)}\left(a_1,a_1,a_2,a_1,a_1,a_4,a_3,a_3;c_3,c_2,c_2,c_1;u,\alpha\ z,y,\alpha\ x\right); \quad (2.45)$$

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_4}\ X_{8}\left(a_1,a_2,a_3;c_1,c_2,c_3;\alpha^2\beta\ x,\alpha\ y,\alpha\ z\right)\left\{\alpha^{a_1-1}\beta^{c_1-1}\right\}$$
$$=\alpha^{a_1-1}\beta^{c_1-1}\ X_{28}^{(4)}\left(a_1,a_1,a_1,a_1,a_1,a_4,a_3,a_2;c_1,c_1,c_3,c_2;\alpha^2\beta\ x,u,\alpha\ z,\alpha\ y\right); \quad (2.46)$$

$$\left(1-u\left[D_{\alpha}^{2}\ \beta^{-1}D_{\beta}^{-1}\ \gamma^{-1}D_{\gamma}^{-1}\alpha^{2}\right]\right)^{-a}\ F_{A}^{(3)}\left(a_1,a_2,a_3,a_4;c_1,c_2,c_3;\alpha\beta\ x,\alpha\ y,\alpha\ z\right)\left\{\alpha^{a_1-1}\ \beta^{c_1-1}\ \gamma^{a-1}\right\}$$
$$=\alpha^{a_1-1}\ \beta^{c_1-1}\ \gamma^{a-1}\ X_{28}^{(4)}\left(a_1,a_1,a_1,a_1,a_1,a_2,a_3,a_4;c_1,c_1,c_2,c_3;u,\alpha\beta\ x,\alpha\ y,\alpha z\right); \quad (2.47)$$

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_5}\ X_{20}\left(a_1,a_2,a_3,a_4;c_1,c_2;\beta\ x,y,\beta\ z\right)\left\{\alpha^{a_6-1}\beta^{c_1-1}\right\}$$
$$=\alpha^{a_6-1}\beta^{c_1-1}\ X_{29}^{(4)}\left(a_1,a_1,a_3,a_5,a_1,a_2,a_4,a_6;c_1,c_2,c_1,c_1;\beta\ x,y,\beta\ z,u\right); \quad (2.48)$$

$$\left(1-u\left[D_\alpha^2\ \beta^{-1}D_\beta^{-1}\ \gamma^{-1}D_\gamma^{-1}\alpha^2\right]\right)^{-a}\ F^{(3)}\begin{bmatrix} -::-;-;-:a_1,a_2;a_3,a_4;a_5,a_6; & \\ & \alpha\ x,\beta\ y,\beta\ z \\ -::-;c_1;-:\quad c_2;\quad -;\quad -; & \end{bmatrix}\left\{\alpha^{a_1-1}\ \beta^{c_1-1}\ \gamma^{a-1}\right\} \tag{2.49}$$
$$=\alpha^{a_1-1}\ \beta^{c_1-1}\ \gamma^{a-1}\ X_{29}^{(4)}\left(a_1,a_1,a_3,a_5,a_1,a_2,a_4,a_6;c_1,c_2,c_1,c_1;u,\alpha\ x,\beta\ y,\beta\ z\right);$$

$$\left(1-u\left[D_\alpha\ \beta^{-1}D_\beta^{-1}\ \alpha\right]\right)^{-a_5}\ X_{18}\left(a_1,a_2,a_3,a_4;c;\beta\ x,\beta\ y,\beta\ z\right)\left\{\alpha^{a_6-1}\beta^{c-1}\right\} \tag{2.50}$$
$$=\alpha^{a_6-1}\beta^{c-1}\ X_{30}^{(4)}\left(a_1,a_1,a_3,a_5,a_1,a_2,a_4,a_6;c,c,c,c;\beta\ x,\beta\ y,\beta\ z,u\right);$$

$$\left(1-u\left[D_\alpha^2\ \beta^{-1}D_\beta^{-1}\ \gamma^{-1}D_\gamma^{-1}\alpha^2\right]\right)^{-a}\ F_B^{(3)}\left(a_1,a_2,a_3,a_4,a_5,a_6;c;\alpha\beta\ x,\beta\ y,\beta\ z\right)\left\{\alpha^{a_1-1}\ \beta^{c-1}\ \gamma^{a-1}\right\} \tag{2.51}$$
$$=\alpha^{a_1-1}\ \beta^{c-1}\ \gamma^{a-1}\ X_{30}^{(4)}\left(a_1,a_1,a_2,a_3,a_1,a_4,a_5,a_6;c,c,c,c;u,\alpha\beta\ x,\ \beta y,\beta\ z\right);$$

where $X_1, X_2, \ldots, X_{20}$ are the Exton functions of three variables (see [8]), H_A, H_B, H_C are Srivastava's functions (see, for details [16] and [17]), $F_A^{(3)}, F_B^{(3)}, F_E, F_F, F_P$ are three variables Lauricella functions (cf. [16,Sections 1.4 and 1.5] and [17, Chapter 1]) and $F^{(3)}[x,y,z]$ is the general triple hypergeometric series (see e.g. [16, p. 44, (14) and (15)].

Proof. For convenience and simplicity, by denoting the left-hand side of (2.1) by I and using the definition of Exton's triple series X_1 and the formulas (1.31) and (1.32), we get

$$I=\left(1-u\left[D_\alpha\ \beta^{-1}D_\beta^{-1}\alpha\right]\right)^{-a_1} X_1\left(a_1,a_2;c_2,c_1;\frac{x}{\sigma^2},\frac{y}{\sigma^2},\frac{\alpha\ z}{\sigma}\right)\left\{\alpha^{a_2-1}\beta^{c_3-1}\right\}$$
$$=\sum_{m,n,p=0}^{\infty}\frac{(a_1)_{2m+2n+p}\,(a_2)_p}{(c_1)_{n+p}\,(c_2)_m\ m!\,n!}x^m\ y^n\ z^p\left(1-u\left[D_\alpha\ \beta^{-1}D_\beta^{-1}\alpha\right]\right)^{-a_1-2m-2n-p}\left\{\alpha^{a_2+p-1}\beta^{c_3-1}\right\}$$
$$=\alpha^{a_2-1}\beta^{c_3-1}\sum_{m,n,p,q=0}^{\infty}\frac{(a_1)_{2m+2n+p+q}\,(a_2)_{p+q}}{(c_1)_{n+p}\,(c_2)_m\,(c_3)_q}\frac{x^m}{m!}\frac{y^n}{n!}\frac{(\alpha\ z)^p}{p!}\frac{u^q}{q!},$$

therefore

$$I=\alpha^{a_2-1}\beta^{c_3-1}\ X_1^{(4)}\left(a_1,a_1,a_1,a_1,a_1,a_1,a_2,a_2;c_2,c_1,c_1,c_3;x\ ,y\ ,\alpha\ z\ ,u\right),$$

which complete the proof of (2.1). Similarly, by applying the same techniques and making use of Exton's functions X_1 to X_{20} Srivastava's functions H_A, H_B, H_C, Lauricella's functions $F_A^{(3)}, F_B^{(3)}, F_E, F_F, F_P$ and $F^{(3)}[x,y,z]$ the general triple hypergeometric series , we can prove the assertions (2.2) to (2.51).

3. Special Cases

Some operational connections can be established in this section as the special cases of the results were obtained in previous section. First, substituting $x=0$ in (2.1), (2.2), (2.5), (2.8), (2.9), (2.11), (2.21), (2.31) and (2.43), after a little simplification, we obtain a number of operational relations among hypergeometric functions of two and three variables

$$\left(1-u\left[D_\alpha\ \beta^{-1}D_\beta^{-1}\ \alpha\right]\right)^{-a_1}\ H_3\left(a_1,a_2;c_1;\frac{y}{\sigma^2},\frac{\alpha\ z}{\sigma}\right)\left\{\alpha^{a_2-1}\beta^{c_3-1}\right\}=\alpha^{a_2-1}\beta^{c_3-1}\ X_3\left(a_1,a_2;c_1,c_3;y,\alpha\ z,u\right); \tag{3.1}$$

$$\left(1-u\left[D_\alpha\ \beta^{-1}D_\beta^{-1}\ \alpha\right]\right)^{-a_1}\ H_4\left(a_2,a_1;c_3,c_2;\alpha^2 z,\frac{\alpha\ y}{\sigma}\right)\left\{\alpha^{a_2-1}\beta^{c_4-1}\right\}=\alpha^{a_2-1}\beta^{c_4-1}\ X_4\left(a_2,a_1;c_3,c_2,c_4;\alpha^2 z,\alpha\ y,u\right); \tag{3.2}$$

$$\left(1-u\left[D_\alpha\ \beta^{-1}D_\beta^{-1}\ \alpha\right]\right)^{-a_1}\ H_3\left(a_2,a_1;c_1;\alpha^2 z,\frac{\alpha\ y}{\sigma}\right)\left\{\alpha^{a_2-1}\beta^{c_2-1}\right\}=\alpha^{a_2-1}\beta^{c_2-1}\ X_3\left(a_2,a_1;c_1,c_2;\alpha^2 z,\alpha\ y,u\right); \tag{3.3}$$

$$\left(1-u\left[D_\alpha\ \beta^{-1}D_\beta^{-1}\ \alpha\right]\right)^{-a_1}\ H_4\left(a_1,a_2;c_2,c_3;\frac{y}{\sigma^2},\frac{z}{\sigma}\right)\left\{\alpha^{a_3-1}\beta^{c_1-1}\right\}=\alpha^{a_3-1}\beta^{c_1-1}\ X_8\left(a_1,a_2,a_3;c_2,c_3,c_1;y,z,u\right); \tag{3.4}$$

$$\left(1-u\left[D_\alpha\ \beta^{-1}D_\beta^{-1}\ \alpha\right]\right)^{-a_1}\ H_4\left(a_1,a_2;c_2,c_3;\frac{y}{\sigma^2},\frac{\beta\ z}{\sigma}\right)\left\{\alpha^{a_3-1}\beta^{c_3-1}\right\}=\alpha^{a_3-1}\beta^{c_3-1}\ X_7\left(a_1,a_2,a_3;c_2,c_3;y,\beta\ z,u\right). \tag{3.5}$$

$$\left(1-u\left[D_{\alpha}^{2}\ \beta^{-1}D_{\beta}^{-1}\ \gamma^{-1}D_{\gamma}^{-1}\alpha^{2}\right]\right)^{-a}\ F_{2}\left(a_{3},a_{2},a_{1};c_{2},c_{3};y,\alpha\ z\right)\left\{\alpha^{a_{1}-1}\ \beta^{c_{4}-1}\ \gamma^{a-1}\right\}$$
$$=\alpha^{a_{1}-1}\ \beta^{c_{4}-1}\ \gamma^{a-1}\ X_{17}\left(a_{1},a_{3},a_{2};c_{4},c_{3},c_{2};u,\alpha\ z,y\right); \tag{3.6}$$

$$\left(1-u\left[D_{\alpha}^{2}\ \beta^{-1}D_{\beta}^{-1}\ \gamma^{-1}D_{\gamma}^{-1}\alpha^{2}\right]\right)^{-a}\ F_{1}\left(a_{3},a_{2},a_{1};c;\beta\ y,\alpha\beta\ z\right)\left\{\alpha^{a_{1}-1}\ \beta^{c-1}\ \gamma^{a-1}\right\}$$
$$=\alpha^{a_{1}-1}\ \beta^{c-1}\ \gamma^{a-1}\ X_{13}\left(a_{1},a_{3},a_{2};c;u,\alpha\beta\ z,\beta\ y\right); \tag{3.7}$$

$$\left(1-u\left[D_{\alpha}^{2}\ \beta^{-1}D_{\beta}^{-1}\ \gamma^{-1}D_{\gamma}^{-1}\alpha^{2}\right]\right)^{-a}\ F_{4}\left(a_{1},a_{3};c_{2},c_{3};\alpha\ y,\alpha\ z\right)\left\{\alpha^{a_{3}-1}\ \beta^{c_{4}-1}\ \gamma^{a-1}\right\}$$
$$=\alpha^{a_{3}-1}\ \beta^{c_{4}-1}\ \gamma^{a-1}\ X_{4}\left(a_{3},a_{1};c_{4},c_{2},c_{3};u,\alpha\ y,\alpha\ z\right); \tag{3.8}$$

$$\left(1-u\left[D_{\alpha}^{2}\ \beta^{-1}D_{\beta}^{-1}\ \gamma^{-1}D_{\gamma}^{-1}\alpha^{2}\right]\right)^{-a}\ F_{3}\left(a_{2},a_{1},a_{3},a_{4};c_{2};\beta\ y,\alpha\beta\ z\right)\left\{\alpha^{a_{1}-1}\ \beta^{c_{2}-1}\ \gamma^{a-1}\right\}$$
$$=\alpha^{a_{1}-1}\ \beta^{c_{2}-1}\ \gamma^{a-1}\ X_{18}\left(a_{1},a_{4},a_{2},a_{3};c_{2};u,\alpha\beta\ z,\beta\ y\right). \tag{3.9}$$

Now, setting $z=0$ in (3.2) and (3.5), we get relationships between Gaussian hypergeometric series ${}_2F_1$ (cf. [16, p.13(1) and 18(17)]) and Appell's series F_4 (see [16, p. 23(5)]), Horn's series H_4 (see [16, p. 24(12)]; also [17, p. 57(28)])

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_{1}}\ {}_{2}F_{1}\left(a_{1},a_{2};c_{2};\frac{\alpha\ y}{\sigma}\right)\left\{\alpha^{a_{2}-1}\beta^{c_{4}-1}\right\}=\alpha^{a_{2}-1}\beta^{c_{4}-1}\ F_{4}\left(a_{1},a_{2};c_{2},c_{4};\alpha\ y,u\right) \tag{3.10}$$

and

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_{1}}\ {}_{2}F_{1}\left(\frac{a_{1}}{2},\frac{a_{1}+1}{2};c_{2};\frac{4y}{\sigma^{2}}\right)\left\{\alpha^{a_{3}-1}\beta^{c_{3}-1}\right\}=\alpha^{a_{3}-1}\beta^{c_{3}-1}\ H_{4}\left(a_{1},a_{3};c_{2},c_{3};y,u\right), \tag{3.11}$$

respectively. If in (3.10) and (3.11), we let $y=0$, we have a known result due to Bin-Saad [1].
On other hand, if we put $x=0$ in (2.10), (2.14), (2.16) and (2.20), then we obtain connections between Appell's series F_1, F_2 (see [16, p. 22(2) and 23(3)]) and Srivastava's series H_A, H_B, H_C

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_{2}}\ F_{2}\left(a_{1},a_{2},a_{3};c_{2},c_{3};\frac{y}{\sigma},\alpha\ z\right)\left\{\alpha^{a_{3}-1}\beta^{c_{4}-1}\right\}=\alpha^{a_{3}-1}\beta^{c_{4}-1}\ H_{B}\left(a_{1},a_{2},a_{3};c_{2},c_{4},c_{3};y,u,\alpha\ z\right); \tag{3.12}$$

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_{2}}\ F_{1}\left(a_{1},a_{2},a_{3};c_{1};\frac{y}{\sigma},\alpha\ z\right)\left\{\alpha^{a_{3}-1}\beta^{c_{3}-1}\right\}=\alpha^{a_{3}-1}\beta^{c_{3}-1}\ H_{A}\left(a_{2},a_{3},a_{1};c_{3},c_{1};u,\alpha\ z,y\right); \tag{3.13}$$

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_{3}}\ F_{2}\left(a_{1},a_{2},a_{3};c_{1},c_{2};\alpha\ y,\frac{\beta\ z}{\sigma}\right)\left\{\alpha^{a_{2}-1}\beta^{c_{2}-1}\right\}=\alpha^{a_{2}-1}\beta^{c_{2}-1}\ H_{A}\left(a_{1},a_{2},a_{3};c_{1},c_{2};\alpha\ y,u,\beta\ z\right); \tag{3.14}$$

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_{2}}\ F_{1}\left(a_{1},a_{2},a_{3};c\,;\frac{\beta\ y}{\sigma},\alpha\beta\ z\right)\left\{\alpha^{a_{3}-1}\beta^{c-1}\right\}=\alpha^{a_{3}-1}\beta^{c-1}\ H_{C}\left(a_{1},a_{2},a_{3};c\,;\beta\ y,u,\alpha\beta\ z\right). \tag{3.15}$$

If in (3.12), we let $z=0$ and in (3.15), we take $z=0$, we find that

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_{2}}\ {}_{2}F_{1}\left(a_{1},a_{2};c_{2};\frac{y}{\sigma}\right)\left\{\alpha^{a_{3}-1}\beta^{c_{4}-1}\right\}=\alpha^{a_{3}-1}\beta^{c_{4}-1}\ F_{2}\left(a_{2},a_{1},a_{3};c_{2},c_{4};y,u\right) \tag{3.16}$$

and

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_{2}}\ {}_{2}F_{1}\left(a_{1},a_{2};c\,;\frac{\beta\ y}{\sigma}\right)\left\{\alpha^{a_{3}-1}\beta^{c-1}\right\}=\alpha^{a_{3}-1}\beta^{c-1}\ F_{1}\left(a_{2},a_{1},a_{3};c\,;\beta\ y,u\right), \tag{3.17}$$

respectively.
Again, if in (2.22), (2.24), (2.26), (2.28), (2.30), (2.32), (2.34) and (2.36), we let $x=0$, yield the known results given by Bin-Saad and Maisoon [2].
Secondly, put $y=0$ in (2.3) and (2.4), we have the elegant formulas

$$\left(1-u\left[D_{\alpha}\ \beta^{-1}D_{\beta}^{-1}\ \alpha\right]\right)^{-a_{1}}\ {}_{2}F_{1}\left(\frac{a_{1}}{2},\frac{a_{1}+1}{2};c_{1};\frac{4x}{\sigma^{2}}\right)\ {}_{2}F_{1}\left(\frac{a_{2}}{2},\frac{a_{2}+1}{2};c_{2};4\alpha^{2}z\right)\left\{\alpha^{a_{2}-1}\beta^{c_{3}-1}\right\}$$
$$=\alpha^{a_{2}-1}\beta^{c_{3}-1}\ X_{12}\left(a_{1},a_{2};c_{1},c_{2},c_{3};x,\alpha^{2}z,u\right) \tag{3.18}$$

and

$$\left(1-u\left[D_\alpha\ \beta^{-1}D_\beta^{-1}\ \alpha\right]\right)^{-a_1} F_3\left(\frac{a_1}{2},\frac{a_2}{2},\frac{a_1+1}{2},\frac{a_2+1}{2};c_1;\frac{4x}{\sigma^2},4\alpha^2 z\right)\left\{\alpha^{a_2-1}\beta^{c_3-1}\right\}$$
$$=\alpha^{a_2-1}\beta^{c_3-1}\ X_{11}\left(a_1,a_2;c_1,c_3;x,\alpha^2 z,u\right), \tag{3.19}$$

respectively, where F_3 is the Appell series (see [16, p. 23(4)]).
Lastly, in (2.12) and (2.32), taking $z=0$, we obtain the following operational connections:

$$\left(1-u\left[D_\alpha\ \beta^{-1}D_\beta^{-1}\ \alpha\right]\right)^{-a_2} H_3\left(a_1,a_2;c_1;x,\frac{y}{\sigma}\right)\left\{\alpha^{a_3-1}\beta^{c_3-1}\right\}$$
$$=\alpha^{a_3-1}\beta^{c_3-1}\ X_{14}\left(a_1,a_2,a_3;c_1,c_3;x,y,u\right) \tag{3.20}$$

and

$$\left(1-u\left[D_\alpha\ \beta^{-1}D_\beta^{-1}\ \alpha\right]\right)^{-a_2} H_4\left(a_1,a_2;c_2,c_1;x,\frac{y}{\sigma}\right)\left\{\alpha^{a_3-1}\beta^{c_3-1}\right\}$$
$$=\alpha^{a_3-1}\beta^{c_3-1}\ X_{17}\left(a_1,a_2,a_3;c_2,c_1,c_3;x,y,u\right), \tag{3.21}$$

respectively.

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