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Research Article

Ideal Convergent Double Sequence Spaces in Random *n*-Normed Spaces

Sabiha Tabassum[1,*] and Mohd Khalid Rafat Khan[2]

[1]Department of Applied Mathematics, Zakir Hussain College Of Engineering and Technology, Aligarh Muslim University, Aligarh-202002, India
[2]College of Science & Theoretical Studies, Saudi Electronic University, Al Muledah Quarter, Buraidha-52571, K.S.A.

* Corresponding author, e-mail: (sabiha.math08@gmail.com)

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Abstract: In this article we define and study the notion of Δ_m^n-Ideal convergent and Δ_m^n-Cauchy double sequence in a random n-normed space $(X, F, *)$ and proved some interesting results.

Keywords: Double Sequence, Ideal, I-convergence, Δ_m^n-Ideal Convergence , *n*-Norm, Random *n*-Norm.

1. INTRODUCTION

A double sequence is denoted by $x = (x_{ij})$. In the case of one variable, we began with the study of sequences of numbers x_i , where the suffix i could be any integer. Here double sequences (see[7]) have a corresponding importance. These are sets of numbers x_{ij} with two subscripts, which run through the sequence of all integers independently of each other, so that we have, for example, the numbers

$$\begin{matrix} x_{11} & x_{12} & x_{13} & \dots \\ x_{21} & x_{22} & x_{23} & \dots \\ x_{31} & x_{32} & x_{33} & \dots \end{matrix}$$

Examples of such sequences are the sets of numbers

1. $x_{ij} = \frac{1}{i+j}$;
2. $x_{ij} = \frac{1}{i^2+j^2}$;
3. $x_{ij} = \frac{i}{i+j}$.

Definition 1.1. [7] A double sequence $x = (x_{jk})$ has pringsheim limit L (denoted by $P-\lim x = L$) provided that given $\varepsilon > 0$ there exists $N \in \mathbb{N}$ such that $|x_{jk} - L| < \varepsilon$ whenever $j, k > N$. We shall describe such an x more briefly as "P-convergent".

A double sequence (x_{jk}) is bounded if

$$\|x\| = \sup_{j,k\geq 0} |x_{jk}| < \infty.$$

Remark 1.1. In contrast to the case for single sequences, a P-convergent double sequences need not be bounded. The initial works on double sequences is found in Bromwich [5], Tripathy [3], Basarir and Solancan [15] and many others.

The concept of statistical convergence was defined by Steinhauss[9] at a conference held at Wroclaw University, poland in 1949 and also independently by fast[8] and Schoenberg [11], Menger [13], M. Gurdal[16], Karakus[21], Buck[18], Connor[12] etc. Statistical convergence is a generalization of the usual notation of convergence that parallels the usual theory of convergence. Kostryko et al.[18] introduced the notion of I-convergence. The notion of I-convergence is a generalization of statistical convergence. It was studied by many authors in [17],[22]. Here we give some preliminaries about the notion of I-convergence.

Definition 1.4. Let X be a non empty set. Then a family of sets $I \subseteq 2^X$ (2^X denoting the power set of X) is said to be an ideal if

1. $\varnothing \in I$
2. I is additive i.e $A, B \in I \Rightarrow A \cup B \in I$
3. I is hereditary i.e $A \in I, B \subseteq A \Rightarrow B \in I$.

An Ideal $I \subseteq 2^X$ is called non-trivial if $I \neq 2^X$. A non-trivial ideal $I \subseteq 2^X$ is called admissible if $\{\{x\} : x \in X\} \subseteq$ I. A non-trivial ideal I is maximal if there cannot exist any non-trivial ideal $J \neq I$ containing I as a subset.

For each ideal I, there is a filter $£(I)$ corresponding to I.

i.e $£(I) = \{K \subseteq I \quad N : K^c \in I\}$,where $K^c = I \quad N - K$.

Definition 1.5. A sequence $(x_{ij}) \in \omega$ is said to be I-convergent to a number L if for every $\varepsilon > 0$, $\{i, j \in I \quad N : |x_{ij} - L| \geq \varepsilon\} \in$ I. In this case we write $I - \lim x_{ij} = L$.

2. MATERIALS AND METHODS

Random 2-normed space and I-convergence

The theory of probabilistic normed spaces was initiated and developed in [8],[9]. This theory is important as a generalization of deterministic results of linear normed spaces and also in the study of random operator equations. Further it was extended to random/probabilistic 2-normed spaces by Golet[10] using the concept of 2-norm of Gähler[20].

Definition 2.1. A function $f:\mathbb{R}\to\mathbb{R}_0^+$ is called a *distribution functions* if it is non-decreasing and left continuous with $\inf_{t\in\mathbb{R}} f(t)=0$ and $\sup_{t\in\mathbb{R}} f(t)=1$. By D^+, we denote the set of all distribution functions such that $f(0)=0$.

If $a\in\mathbb{R}_0^+$, then $H_\alpha\in D^+$, where $H_a(t)=\begin{cases}1, & if\ t>a;\\ 0, & if\ t\le a.\end{cases}$

It is obvious that $H_0\ge f$ for all $f\in D^+$.

A *t-norm* is a continuous mapping $*:[0,1]\times[0,1]\to[0,1]$ such that $([0,1],*)$ is an Abelian monoid with unit one and $c*d\ge a*b$ if $c\ge a$ and $d\ge b$ for all $a,b,c,d\in[0,1]$. A triangle function τ is a binary operation on D^+, which is commutative, associative and $t(f,H_0)=f$ for every $f\in D^+$.

In [20], Gähler introduced the following concept of a 2-normed space in the mid of 1960's. Since then, many researchers have studied this concept and obtained various results.

Definition 2.2. Let X be a real vector space of dimension $d>1$. A real-valued function $\|.,.\|$ from X^2 into $\mathbb{R}$ satisfying the following conditions:

1. $\|x_1,x_2\|=0$ if and only if x_1,x_2 are linearly dependent,
2. $\|x_1,x_2\|$ is invariant under permutation,
3. $\|\alpha x_1,x_2\|=|\alpha|\|x_1,x_2\|$ for any $\alpha\in\mathbb{R}$,
4. $\|x+x',x_2\|\le\|x,x_2\|+\|x',x_2\|$

is called a 2-norm on X and the pair $(X,\|.,.\|)$ is called a 2-normed space.

A trivial example of a 2-normed space is $X=R^2$, equipped with the Euclidean 2-norm $\|x_1,x_2\|=$ the area of the parallelogram spanned by the vectors x_1,x_2 which may be given explicitly by the formula

$$\| x_1, x_2 \|_E = | det(x_{ij} | = abs(det(< x_i, x_j >)),$$

where $x_i = (x_{i1}, x_{i2}) \in R^2$ for each $i = 1,2$.

Let $n \in \mathbb{N}$ and X be a real vector space of dimension d, where $n \le d$. A real valued function $\|.,...,.\|$ on X^n satisfying the following four conditions:

1. $\| x_1, x_2, ..., x_n \| = 0$ if and only if $x_1, x_2, ..., x_n$ are linearly dependent;
2. $\| x_1, x_2, ..., x_n \|$ is invariant under permutation:
3. $\| \alpha x_1, x_2, ..., x_n \| = \alpha \| x_1, x_2, ..., x_n \|$, for any $\alpha \in R$:
4. $\| x + x', x_2, ..., x_n \| \le \| x, x_2, ..., x_n \| + \| x', x_2, ..., x_n \|$

is called an $n-norm$ on X, and the pair $(X, \|.,.,...,.\|)$ is then called an n-normed space.

As a standard example of a n-normed space we may take R^n being equipped with the n-norm $\| x_1, x_2, ..., x_n \|_E =$ the volume of the n-dimensional parallelopiped spaned by the vectors $x_1, x_2, ..., x_n$ which may be given explicitly by the formula

$$\| x_1, x_2, ..., x_n \|_E = | \det(x_{ij}) |,$$

where $x_i = (x_{i1}, x_{i2}, ..., x_{in}) \in R^n$ for each $i = 1,2,...,n.$

Recently, Golet [10] used the idea of a 2-normed space to define a random 2-normed space since then many researchers have studied this concept for instance [2],[14], [22].

Definition 2.3. [6] Let X be a linear space of dimension $d > 1, \tau$ a triangle, and $F : X \times X \to D^+$. Then F is called a probabilistic 2-norm and (X, F, τ) a probabilistic 2-normed space if the following conditions are satisfied:

1. $F(x, y; t) = H_0(t)$ if x and y are linearly dependent, where $F(x, y; t)$ denotes the value of $F(x, y)$ at $t \in R$,
2. $F(x, y; t) \ne H_0(t)$ if x and y are linearly independent,
3. $F(x, y; t) = F(y, x; t)$, for all $x, y \in X$,
4. $F(\alpha x, y; t) = F(x, y; \frac{t}{\alpha})$, for every $t > 0, \alpha \ne 0$ and $x, y \in X$,
5. $F(x + y, z; t) \ge \tau(F(x, z; t), F(y, z; t))$, whenever $x, y, z \in X$. If 5 is replaced by
6. $F(x + y, z; t_1 + t_2) \ge F(x, z; t_1) * Fy, z; t_2)$, for all $x, y, z \in X$ and $t_1, t_2 \in R_0^+$;

then $(X, F, *)$ is called a random 2-normed space (for short, R2NS).

Remark 2.1. Every 2-normed space can be made a random 2-normed space in a natural way by setting $F(x, y; t) = H_0(t - \|x, y\|)$ for every $x, y \in X, t > 0$ and $a * b = \min\{a, b\}, a, b \in [0,1]$.

Example 2.1. Let $(X,\|.,.\|)$ be a 2-normed space with $\|x,z\|=\|x_1z_2-x_2z_1\|$, $x=(x_1,x_2)$, $z=(z_1,z_2)$ and $a*b=ab, a,b\in[0,1]$. For all $x\in X, T>0$ and nonzero $z\in X$, consider

$$F(x,y;t)=\begin{cases}\frac{t}{t+\|x,z\|}, & if\ t>0;0, \qquad if\ t\le 0.\end{cases}$$

Then $(X,F,*)$ is a random 2-normed space.

Definition 2.4. Let X be a linear space of dimension $d>n$, and let F be a mapping defined on the cartesian product of X by itself of n times X^n into D^+ such that the following conditions are satisfied:

1. $F(x_1,x_2,...,x_n;t)$ is invariant under any permutation of $x_1,x_2,...,x_n\in X$
2. $F(x_1,x_2,...,x_n;t)=F(x_1,x_2,...,x_n;\frac{t}{\varphi(\alpha)})$ for every $x_1,x_2,...,x_n\in X$ and $\alpha\in R$
3. $F(x_1,x_2,...,x_{n-1},y+z;t)\ge\tau(F(x_1,x_2,...,x_{n-1},y;t),F(x_1,x_2,...,x_{n-1},z;t))$, for every $x_1,x_2,...,x_{n-1},y,z\in X$
4. $F(x_1,x_2,...,x_n;t_1+t_2)\ge F(x_1,x_2,...,x_n;t_1)*F(x_1,x_2,...,x_n;t_2))$, for all $x_1,x_2,...,x_n\in X$

then $(X,F,*)$ is called a probabilistic n-normed space (for short, RnNS).

Definition 2.5. A sequence $x=(x_{k,l})$ in a random 2-normed space $(X,F,*)$ is said to be *double convergent* (or F-convergent) to $l\in X$ with respect to F if for each $\varepsilon>0,\eta\in(0,1)$, there exists a positive integer n_0 such that $F(x_{kl}-l,z;\varepsilon)>1-\eta$, whenever $k,l\ge n_0$ and for nonzero $z\in X$. In this case we write $F-\lim_{k,l}x_{kl}=l$, and l is called the F-limit of $x=(x_{k,l})$.

Definition 2.6. A sequence $x=(x_{k,l})$ in a random 2-normed space $(X,F,*)$ is said to be *double Cauchy* with respect to F if for each $\varepsilon>0,\eta\in(0,1)$, there exists $N=N(\varepsilon)$ and $M=M(\varepsilon)$ such that $F(x_{kl}-x_{pq},z;\varepsilon)>1-\eta$, whenever $k,p\ge N$ and $l,q\ge M$ for nonzero $z\in X$.

Definition 2.7. A sequence $x=(x_{k,l})$ in a random 2-normed space $(X,F,*)$ is said to be *double statistically convergent or* S^{2R2N}*-convergent* to some $l\in X$ with respect to F if for each $\varepsilon>0\,\eta\in(0,1)$, and for nonzero $z\in X$. such that

$$\delta(\{(k,l)\in N\times N: F(x_{kl}-l,z;\varepsilon)\le 1-\eta\})=0.$$

In other words, we can write the sequence (x_{kl}) *double statistically converges to* l in R2N space $(X,F,*)$ if

$$\lim_{m,n\to\infty}\frac{1}{mn}|\{k\le m,l\le n: F(x_{kl}-l,z;\varepsilon)\le 1-\eta\}|=0.$$

or equivalently,

$$\delta(\{k,l\in N: F(x_{kl}-l,z;\varepsilon)>1-\eta\})=1,$$

that is

$$S^2 - \lim_{k,l\to\infty} F(x_{kl} - l, z; \varepsilon) = 1$$

In this case we write $S^{2R2N} - \lim x = l$, and l is called the S^{2R2N}-limit of x. Let $S^{2R2N}(X)$ denote the set of all double statistically convergent sequences in a random 2-normed space $(X, F, *)$.

Definition 2.8. Let I be a nontrivial ideal. A double sequence $x = (x_{kl})$ is said to be $I-convergent$ in $(X, F, *)$ or simply $I_F - convergent$ to l if for every $\varepsilon > 0$, $\eta \in (0,1)$ and nonzero $z \in X$, we have $\{k, l \in N : F(x_{kl} - l, z; \varepsilon) \le 1 - \eta\} \in I$.

3. RESULTS AND DISCUSSION

Random n-normed space and I-convergence

Definition 3.1. A sequence $x = (x_{j,k})$ in a random n-normed space $(X, F, *)$ is said to be *double convergent* (or F-convergent) to $l \in X$ with respect to F if for each $\varepsilon > 0, \eta \in (0,1)$, there exists a positive integer n_0 such that $F(x_{jk} - l, z_1, z_2, .., z_{n-1}; \varepsilon) > 1 - \eta$, whenever $j, k \ge n_0$ and for nonzero $z_1, z_2, .., z_{n-1} \in X$. In this case we write $F - \lim_{j,k} x_{jk} = l$, and l is called the F-limit of $x = (x_{jk})$.

Definition 3.2. A sequence $x = (x_{jk})$ in a random n-normed space $(X, F, *)$ is said to be *double Cauchy* with respect to F if for each $\varepsilon > 0, \eta \in (0,1)$, there exists $N = N(\varepsilon)$ and $M = M(\varepsilon)$ such that $F(x_{jk} - x_{pq}, z_1, z_2, .., z_{n-1}; \varepsilon) > 1 - \eta$, whenever $j, p \ge N$, $k, q \ge M$ and for nonzero $z_1, z_2, .., z_{n-1} \in X$.

4. MAIN RESULTS

4.1. Δ_m^n-Ideal Convergence

Recently B.Hazarika[4] studied the concept of Δ^n-Ideal convergence and Δ^n-Ideal Cauchy Double sequences in random 2- normed spaces and proved some interesting theorem. In this section we define Δ_m^n-Ideal Convergent sequences in the random n-normed $(X, F, *)$. Also we obtained some basic properties of this notion in random n-normed space.

Definition 4.1. A sequence $x = (x_{jk})$ in a random n-normed space $(X, F, *)$ is said to be Δ_m^n-convergent to $l \in X$ with respect to F if for each $\varepsilon > 0, \eta \in (0,1)$, there exists a positive integer n_0 such that $F(\Delta_m^n x_{jk} - l, z_1, z_2, .., z_{n-1}; \varepsilon) > 1 - \eta$, whenever $j, k \ge n_0$ and for nonzero $z_1, z_2, .., z_{n-1} \in X$. In this case we write $F - \lim_{j,k} \Delta_m^n x_{jk} = l$.

Definition 4.2. A sequence $x=(x_{jk})$ in a random n-normed space $(X,F,*)$ is said to be Δ_m^n-Cauchy with respect to F if for each $\varepsilon>0,\eta\in(0,1)$, there exists a positive integer $M=M(\varepsilon,z_1,z_2,..,z_{n-1})$ and $N=N(\varepsilon,z_1,z_2,..,z_{n-1})$ such that $F(\Delta_m^n x_{jk}-\Delta_m^n x_{pq},z_1,z_2,..,z_{n-1};\varepsilon)>1-\eta$, whenever $j,p\geq M$ and $k,q\geq N$ and for nonzero $z_1,z_2,..,z_{n-1}\in X$.

Definition 4.3. A sequence $x=(x_{jk})$ in a random n-normed space $(X,F,*)$ is said to be Δ_m^n-I- convergent to $l\in X$ with respect to F if for each $\varepsilon>0,\eta\in(0,1)$ and nonzero $z_1,z_2,..,z_{n-1}\in X$ such that

$$\{(j,k)\in N\times N: F(\Delta_m^n x_{jk}-l,z_1,z_2,..,z_{n-1};\varepsilon)\leq 1-\eta\}\in I$$

or equivalently

$$\{(j,k)\in N\times N: F(\Delta_m^n x_{jk}-l,z_1,z_2,..,z_{n-1};\varepsilon)>1-\eta\}\in F.$$

Definition 4.4. A sequence $x=(x_{jk})$ in a random n-normed space $(X,F,*)$ is said to be Δ_m^n-I -Cauchy with respect to F if for each $\varepsilon>0,\eta\in(0,1)$, and nonzero $z_1,z_2,..,z_{n-1}\in X$ there exist $M=M(\varepsilon,z_1,z_2,..,z_{n-1})$ and $N=N(\varepsilon,z_1,z_2,..,z_{n-1})$ such that

$$\{(j,k)\in N\times N: F(\Delta_m^n x_{jk}-\Delta_m^n x_{pq},z_1,z_2,..,z_{n-1};\varepsilon)\leq 1-\eta\}\in I,$$

whenever $j,p\geq N$, $k,q\geq M$,

or equivalently

$$\{(j,k)\in N\times N: F(\Delta_m^n x_{jk}-\Delta_m^n x_{pq},z_1,z_2,..,z_{n-1};\varepsilon)>1-\eta\}\in F.$$

Theorem 4.1. Let $(X,F,*)$ be a random n-normed space.If $x=(x_{jk})$ is a double sequence in X such that $I^{2RnN}-\lim\Delta_m^n x_{jk}=l$ exists, then it is unique.

Proof. Suppose that there exist elements $l_1,l_2(l_1\neq l_2)$ in X such that $I^{2RnN}-\lim_{j,k\to\infty}\Delta_m^n x_{jk}=l_1$ and $I^{2RnN}-\lim_{j,k\to\infty}\Delta_m^n x_{jk}=l_2$.

Let $\varepsilon>0$ be given. Choose $a>0$ such that

$$(1-a)(1+a)>1-\varepsilon. \qquad [4.1]$$

Then for any $t>0$ and for non zero $z_1,z_2,..,z_{n-1}\in X$ we define

$$K_1(a,t)=\{(j,k)\in N\times N: F(\Delta_m^n x_{jk}-l_1,z_1,z_2,..,z_{n-1};\frac{t}{2})\leq 1-a\},$$

$$K_2(a,t)=\{(j,k)\in N\times N: F(\Delta_m^n x_{jk}-l_2,z_1,z_2,..,z_{n-1};\frac{t}{2})\leq 1-a\}.$$

Since $I^{2RnN}-\lim_{j,k\to\infty}\Delta_m^n x_{jk}=l_1$ and $I^{2RnN}-\lim_{j,k\to\infty}\Delta_m^n x_{jk}=l_2$ we have $K_1(a,t)$ and $K_2(a,t)\in I$ for all $t\geq 0.$ Now let $K(a,t)=K_1(a,t)\cup K_2(a,t),$ then it is easy to observe that $K(a,t)\in I$. But we have $K^c(a,t)\in F$. Now if $(j,k)\in K^c(a,t),$ then we have

$$F(l_1-l_2,z_1,z_2,..,z_{n-1},t)\geq F(\Delta_m^n x_{jk}-l_1,z_1,z_2,..,z_{n-1};\frac{t}{2})*F(\Delta_m^n x_{jk}-l_2,z_1,z_2,..,z_{n-1};\frac{t}{2})$$

$$>(1-a)*(1-a).$$

It follows from [4.1] that

$$F(l_1-l_2,z_1,z_2,..,z_{n-1},t)=0$$

for all $t>0$ and non zero $z_1,z_2,..,z_{n-1}\in X.$ Hence $l_1=l_2.$

Theorem 4.2. Let $(X,F,*)$ be a random n-normed space and $x=(x_{jk})$ and $y=(y_{jk})$ be two double sequences in X .

1. If $I^{2RnN}-\lim_{j,k\to\infty}\Delta_m^n x_{jk}=l$ and $c(\neq 0)\in \mathrm{R},$ then $I^{2RnN}-\lim_{j,k\to\infty}c\Delta_m^n x_{jk}=cl.$

2. If $I^{2RnN}-\lim_{j,k\to\infty}\Delta_m^n x_{jk}=l_1$ and $I^{2RnN}-\lim_{j,k\to\infty}\Delta_m^n y_{jk}=l_2$ then $I^{2RnN}-\lim_{j,k\to\infty}\Delta_m^n(x_{jk}+y_{jk})=l_1+l_2.$

Proof of the theorem is straightforward, thus omitted.

Theorem 4.3. Let $(X,F,*)$ be a random n-normed space and $x=(x_{jk})$ be double sequences in X such that $F\lim\Delta_m^n x_{jk}=l,$ then $I^{2RnN}-\lim\Delta_m^n x_{jk}=l.$

Proof Let $F\lim\Delta_m^n x_{jk}=l.$ Then for every $0<\varepsilon<1,t>0$ and non zero $z_1,z_2,..,z_{n-1}\in X,$ there is a positive integer $M=M(\varepsilon,z_1,z_2,..,z_{n-1})$ and $N=N(\varepsilon,z_1,z_2,..,z_{n-1})$ such that

$$F(\Delta_m^n x_{jk}-l,z_1,z_2,..,z_{n-1};t)>1-\varepsilon,$$

for all $j\geq M$ and $k\geq N.$ Since the set

$$K(\varepsilon,t)=\{(j,k)\in N\times N: F(\Delta_m^n x_{jk}-l,z_1,z_2,..,z_{n-1};t)\leq 1-\varepsilon\}$$

$$\subset N\times N-\{(i_{j+1},i_{k+1}),(i_{j+2},i_{k+2}),...\}.$$

Also since I is an admissible ideal, and consequently we have $K(\varepsilon,t)\in I.$ This shows that $I^{2RnN}-\lim\Delta_m^n x_{jk}=l.$

Theorem 4.4. Let $(X, F, *)$ be a random n-normed space and $x = (x_{jk})$ be double sequences in X then $I^{2RnN} - \lim \Delta_m^n x_{jk} = l$ if and only if there exists a subset $K \subseteq N \times N$ such that $K \in F$ and $F - \lim \Delta_m^n x_{jk} = l.$

Proof. Suppose first that $I^{2RnN} - \lim \Delta_m^n x_{jk} = l.$ Then for any $t > 0$, r=1,2,3,... and nonzero $z_1, z_2,.., z_{n-1} \in X$, let

$$B(r,t) = \{(j,k) \in N \times N : F(\Delta_m^n x_{jk} - l, z_1, z_2,.., z_{n-1}; t) > 1 - \frac{1}{r}\}$$

$$K(r,t) = \{(j,k) \in N \times N : F(\Delta_m^n x_{jk} - l, z_1, z_2,.., z_{n-1}; t) \le 1 - \frac{1}{r}\}$$

Since $I^{2RnN} - \lim \Delta_m^n x_{jk} = l$ it follows that $K(r,t) \in I.$

Now for $t > 0$ and $r = 1,2,3,...$ we observe that

$$B(r,t) \supset B(r+1,t)$$

and

$$B(r,t) \in F. \qquad [4.2]$$

Now we have to show that, for $k \in B(r,t), F \lim \Delta_m^n x_{jk} = l.$ Suppose that for $k \in B(r,t), (x_{jk})$ is not convergent to l with respect to $F.$ Then there exists some $s > 0$ such that

$$\{(j,k) \in N \times N : F(\Delta_m^n x_{jk} - l, z_1, z_2,.., z_{n-1}; t) \le 1 - s\}$$

and $s > \frac{1}{r}, r = 1,2,3,..$

Then we have $B(s,t) \in I.$

Furthermore, $B(r,t) \subset B(s,t)$ implies that $B(r,t) \in I$, which contradicts [4.2] as $B(r,t) \in F.$ Hence $F - \lim \Delta_m^n x_{jk} = l.$

Conversely, suppose that there exists a subset $K \subseteq N \times N$ such that $K \in F$ and $F \lim \Delta_m^n x_{jk} = l.$ Then for everY $0 < \varepsilon < 1, t > 0$ and non zero $z_1, z_2,.., z_{n-1} \in X$, we can find out a positive integer $M = M(\varepsilon, z_1, z_2,.., z_{n-1})$ such that

$$F(\Delta_m^n x_{jk} - l, z_1, z_2,.., z_{n-1}; t) > 1 - \varepsilon$$

for all $j,k \ge M$. If we take

$$K(\varepsilon,t) = \{(j,k) \in N \times N : F(\Delta_m^n x_{jk} - l, z_1, z_2,.., z_{n-1}; t) < 1 - \varepsilon\}$$

then it is easy to see that

$$K(\varepsilon,t) \subset N \times N - \{(n_{j+1}, n_{k+1}), (n_{j+2}, n_{k+2}),...\}$$

and since I is a admissible ideal, consequently $K(\varepsilon,t) \in I.$

Hence $I^{2RnN} - \lim \Delta_m^n x_{jk} = l.$

Theorem 4.5. Let $(X,F,*)$ be a random n-normed space. Then $x=(x_{jk})$ in X then $\Delta_m^n - I -$ convergent if and only if it is $\Delta_m^n - I -$ Cauchy.

Proof Let (x_{jk}) be a $\Delta_m^n - I -$ convergent sequence in X. We assume that $I^{2RnN} - \lim \Delta_m^n x_{jk} = l.$ Let $\varepsilon > 0$ be given. Choose $a > 0$ such 4.1 is satisfied. For $t > 0$ and non zero $z_1, z_2, .., z_{n-1} \in X$ define

$$A(a,t) = \{(j,k) \in N \times N : F(\Delta_m^n x_{jk} - l, z_1, z_2, .., z_{n-1}; \frac{t}{2}) \le 1-a\}.$$

Then

$$A^c(a,t) = \{(j,k) \in N \times N : F(\Delta_m^n x_{jk} - l, z_1, z_2, .., z_{n-1}; \frac{t}{2}) > 1-a\}.$$

Since $I^{2RnN} - \lim \Delta_m^n x_{jk} = l$ it follows that $A(a,t) \in I$ and consequently $A^c(a,t) \in F$. Let $p,q \in A^c(a,t)$. Then

$$F(\Delta_m^n x_{jk} - l, z_1, z_2, .., z_{n-1}; \frac{t}{2}) > 1-a. \quad [4.3]$$

If we take

$$B(\varepsilon,t) = \{(j,k) \in N \times N : F(\Delta_m^n x_{jk} - \Delta_m^n x_{pq}, z_1, z_2, .., z_{n-1}; t) \le 1-\varepsilon\},$$

then to prove the result it is sufficient to prove that $B(\varepsilon,t) \subseteq A(a,t)$. Let $(j,k) \in B(\varepsilon,t) \cap A^c(a,t)$, then for non zero $z_1, z_2, .., z_{n-1} \in X$.

$$F(\Delta_m^n x_{jk} - \Delta_m^n x_{pq}, z_1, z_2, .., z_{n-1}; t) \le 1-\varepsilon$$

and

$$F(\Delta_m^n x_{jk} - l, z_1, z_2, .., z_{n-1}; \frac{t}{2}) > 1-a \quad [4.4]$$

Then by definition 4.1, 4.3 and 4.4 we get

$$\begin{aligned} 1-\varepsilon &\ge F(\Delta_m^n x_{jk} - \Delta_m^n x_{pq}, z_1, z_2, .., z_{n-1}; t) \\ &\ge F(\Delta_m^n x_{jk} - l, z_1, z_2, .., z_{n-1}; \frac{t}{2}) * F(\Delta_m^n x_{pq} - l, z_1, z_2, .., z_{n-1}; \frac{t}{2}) \\ &> (1-a) * 1-a) > (1-\varepsilon) \end{aligned}$$

which is not possible. Thus $B(\varepsilon,t) \subset A(r,t)$. Since $A(a,t) \in I$, it follows that $B(\varepsilon,t) \in I$. This shows that (x_{jk}) is $\Delta_m^n - I -$ Cauchy.

Conversely, suppose (x_{jk}) is $\Delta_m^n - I -$ Cauchy but not $\Delta_m^n - I -$ convergent. Then there exists positive integer p,q and non zero $z_1, z_2, .., z_{n-1} \in X$ such that.

$$A(\varepsilon,t) = \{(j,k) \in N \times N : F(\Delta_m^n x_{jk} - \Delta_m^n x_{pq}, z_1, z_2, .., z_{n-1}; t) \le 1-\varepsilon\}.$$

Hence, $A(\varepsilon,t) \in I$, so we have

$$A^c(\varepsilon,t) \in F. \quad [4.5]$$

For any $a > 0$ such that 4.1 is satisfied,we take

$$B(a,t) = \{(j,k) \in N \times N : F(\Delta_m^n x_{jk} - l, z_1, z_2, .., z_{n-1}; \frac{t}{2}) > 1-a\}.$$

If $(p,q) \in B(r,t)$, then

$$F(\Delta_m^n x_{pq} - l, z_1, z_2, .., z_{n-1}; \frac{t}{2}) > 1 - a.$$

Since $F(\Delta_m^n x_{jk} - \Delta_m^n x_{pq}, z_1, z_2, .., z_{n-1}; t)$

$$\geq F(\Delta_m^n x_{jk} - l, z_1, z_2, .., z_{n-1}; \frac{t}{2}) * F(\Delta_m^n x_{pq} - l, z_1, z_2, .., z_{n-1}; \frac{t}{2})$$

$$> (1-a) * (1-a) > 1 - \varepsilon,$$

we have

$$\{(j,k) \in N \times N : F(\Delta_m^n x_{jk} - \Delta_m^n x_{pq}, z_1, z_2, .., z_{n-1}; t) \leq 1 - \varepsilon\} \in I,$$

that is, $A^c(\varepsilon, t) \in I,$ which contradicts [4.5]. Hence (x_{jk}) is $\Delta_m^n - I -$ convergent.

Combining Theorem 4.4 and 4.5 we get the following.

Corollary 4.6. Let $(X, F, *)$ be a random n-normed space. Then $x = (x_{jk})$ be a double sequence in X. Then, the following statements are equivalent:

1. x is $\Delta_m^n - I -$ convergent,
2. x is $\Delta_m^n - I -$ Cauchy, and
3. there exists a subset $K \subseteq \mathbb{N} \times \mathbb{N}$ such that $K \in F$ and $F - \lim \Delta_m^n x_{jk} = l.$

Theorem 4.7. Let $(X, F, *)$ be a random n-normed space and $x = (x_{jk})$ be a double sequence in X. Let I be a non trivial ideal in $\mathbb{N}$. If there is a $\Delta_m^n - I -$ convergent sequence $y = (y_{jk})$ in X such that $\{(j,k) \in N \times N : F(\Delta_m^n y_{jk} \neq \Delta_m^n x_{jk}\} \in I,$ then x is also $\Delta_m^n - I -$convergent in X.

Proof. Suppose that $\{(j,k) \in N \times N : F(\Delta_m^n y_{jk} \neq \Delta_m^n x_{jk}\} \in I,$ and $I^{2RnN} - \lim \Delta_m^n y_{jk} = l.$ Let $0 < \varepsilon < 1$ be given. Then, for $t > 0$ and non-zero $z_1, z_2, .., z_{n-1} \in X,$ we get

$$\{(j,k) \in N \times N : F(\Delta_m^n y_{jk} - l, z_1, z_2, .., z_{n-1}; \frac{t}{2}) \leq 1 - \varepsilon\} \in I.$$

for every $0 < \varepsilon < 1,$

$$\{(j,k) \in N \times N : F(\Delta_m^n x_{jk} - l, z_1, z_2, .., z_{n-1}; \frac{t}{2}) \leq 1 - \varepsilon\}$$

$$\subseteq \{(j,k) \in N \times N : F(\Delta_m^n y_{jk} \neq \Delta_m^n x_{jk}\}$$

$$\cup \{(j,k) \in N \times N : F(\Delta_m^n y_{jk} - l, z_1, z_2, .., z_{n-1}; \frac{t}{2}) \leq 1 - \varepsilon\}. \qquad [4.6]$$

As both the sets of right hand side of [4.6] are in $I,$ we have that

$$\{(j,k) \in N \times N : F(\Delta_m^n x_{jk} - l, z_1 - l_2, z_1, z_2, .., z_{n-1}; \frac{t}{2}) \leq 1 - \varepsilon\} \in I.$$

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