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Research Article

On triple sequence spaces of sliding window rough statistical convergence for measurable function of probability defined by Musielak-Orlicz function

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Abstract: We introduced the triple sequence spaces of sliding window rough statistical convergence for measurable function of probability defined by Musielak-Orlicz function and discuss general properties of among these sequence spaces.

Keywords: Sliding window method, measurable functions, rough statistical convergence, triple sequences, Musielak-Orlicz function.

INTRODUCTION

The idea of rough convergence was introduced by Phu [11], who also introduced the concepts of rough limit points and roughness degree. The idea of rough convergence occurs very naturally in numerical analysis and has interesting applications. Aytar [1] extended the idea of rough convergence into rough statistical convergence using the notion of natural density just as usual convergence was extended to statistical convergence. Pal et al. [10] extended the notion of rough convergence using the concept of ideals which automatically extends the earlier notions of rough convergence and rough statistical convergence.

A triple sequence (real or complex) can be defined as a function $x : \mathbb{N} \times \mathbb{N} \times \mathbb{N} \to \mathbb{R}(\mathbb{C})$, where $\mathbb{N}, \mathbb{R}$ and $\mathbb{C}$ denote the set of natural numbers, real numbers and complex numbers respectively. The different types of notions of triple sequence was introduced and investigated at the initial by *Sahiner et al. [12,13]*, *Esi et al. [2-4]*, *Datta et al. [5]*,*Subramanian et al. [14]*, *Debnath et al. [6]* and many others.

A triple sequence $x = (x_{mnk})$ is said to be triple analytic if

$$sup_{m,n,k} \left| x_{mnk} \right|^{\frac{1}{m+n+k}} < \infty.$$

The space of all triple analytic sequences are usually denoted by Λ^3. A triple sequence $x = (x_{mnk})$ is called triple gai sequence if

$$\left((m+n+k)!\left|x_{mnk}\right|\right)^{\frac{1}{m+n+k}} \to 0 as m,n,k \to \infty.$$

The space of all triple gai sequences are usually denoted by χ^3.

In this paper we denote (γ,η) as a sliding window pair provided:

(i) γ and η are both nondecreasing nonnegative real valued measurable functions defined on $[0,\infty)$,

(ii) $\gamma(\alpha) < \eta(\alpha)$ for every positive real number α, and $\eta(\alpha) \to \infty$ as $\alpha \to \infty$,

(iii) $liminf_{abc}\left(\eta(\alpha) - \gamma(\alpha)\right) > 0$ and

(iv) $(0,\infty] = \bigcup\left\{\left(\gamma(s) - \eta(s)\right] : s \leq \alpha\right\}$ for all $\alpha > 0$.

Suppose $I_{abc} = \left(\gamma(\alpha), \eta(\alpha)\right]$ and $\eta(\alpha) - \gamma(\alpha) = \mu(I_{abc})$, where $\mu(A)$ denotes the Lebesgue measure of the set A.

2 Definitions and Preliminaries

Definition 2.1 *An Orlicz function ([see [7]) is a function* $M : [0,\infty) \to [0,\infty)$ *which is continuous, non-decreasing and convex with* $M(0) = 0,\quad M(x) > 0,$ *for* $x > 0$ *and* $M(x) \to \infty$ *as* $x \to \infty$. *If convexity of Orlicz function* M *is replaced by* $M(x+y) \leq M(x) + M(y)$, *then this function is called modulus function.*

Lindenstrauss and Tzafriri ([8]) used the idea of Orlicz function to construct Orlicz sequence space.

A sequence $g = (g_{mn})$ defined by

$$g_{mn}(v) = sup\left\{|v|u - (f_{mnk})(u) : u \geq 0\right\}, m,n,k = 1,2,\cdots$$

is called the complementary function of a Musielak-Orlicz function f. For a given Musielak-Orlicz function f, [see [9]] the Musielak-Orlicz sequence space t_f is defined as follows

$$t_f = \left\{x \in w^3 : I_f\left(\left|x_{mnk}\right|\right)^{1/m+n+k} \to 0 \quad as \quad m,n,k \to \infty\right\},$$

where I_f is a convex modular defined by

$$I_f(x) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty} f_{mnk}\left(\left|x_{mnk}\right|\right)^{1/m+n+k}, x = (x_{mnk}) \in t_f.$$

We consider t_f equipped with the Luxemburg metric

$$d(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty} f_{mnk}\left(\frac{\left|x_{mnk}\right|^{1/m+n+k}}{mnk}\right)$$

is an extended real number.

Definition 2.2 *Let* X, Y *be a real vector space of dimension w, where* $n \leq m$. *A real valued function* $d_p(x_1, \ldots, x_n) = \|(d_1(x_1,0), \ldots, d_n(x_n,0))\|_p$ *on* X *satisfying the following four conditions:*

(i) $\|(d_1(x_1,0), \ldots, d_n(x_n,0))\|_p = 0$ if and only if $d_1(x_1,0), \ldots, d_n(x_n,0)$ are linearly dependent,

(ii) $\|(d_1(x_1,0), \ldots, d_n(x_n,0))\|_p$ is invariant under permutation,

(iii) $\|(\alpha d_1(x_1,0), \ldots, d_n(x_n,0))\|_p = |\alpha| \|(d_1(x_1,0), \ldots, d_n(x_n,0))\|_p, \alpha \in \mathbb{R}$

(iv)
$d_p\left((x_1,y_1),(x_2,y_2)\cdots(x_n,y_n)\right) = \left(d_X(x_1,x_2,\cdots x_n)^p + d_Y(y_1,y_2,\cdots y_n)^p\right)^{1/p}$ *for* $1 \leq p < \infty$; (or)

(v) $d\left((x_1,y_1),(x_2,y_2),\cdots(x_n,y_n)\right) := \sup\left\{d_X(x_1,x_2,\cdots x_n), d_Y(y_1,y_2,\cdots y_n)\right\}$,

for $x_1, x_2, \cdots x_n \in X, y_1, y_2, \cdots y_n \in Y$ is called the p product metric of the Cartesian product of n metric spaces (see [15]) .

Definition 2.3 *Let* $\eta = (\lambda_{abc})$ *be a non-decreasing sequence of positive real numbers tending to infinity and* $\lambda_{111} = 1$ *and* $\lambda_{a+b+c+3} \leq \lambda_{a+b+c+3} + 1$, *for all* $a, b, c \in \mathbb{N}$. *The collection of all such triple sequences* λ *is denoted by* $\Im$.

The generalized de la Vall`e e-Poussin means are defined by

$$t_{abc}(x) = \lambda_{abc}^{-1} \sum_{m,n,k \in I_{abc}} x_{mnk},$$

where $I_{abc} = [abc - \lambda_{abc} + 1, abc]$. A sequence $x = (x_{mnk})$ is said to $(V, \lambda)-$ summable to a number L if $t_{abc}(x) \to L$, as $abc \to \infty$.

Definition 2.4 *A triple sequence spaces of* (X_{mnk}) *is said to strong* (V, λ) *summable (or shortly :* $[V, \lambda]-$ *convergent to* $\overline{0}$ *if*

$$\lim_{abc \to \infty} \frac{1}{\lambda_{abc}} \sum_{m \in I_a} \sum_{n \in I_b} \sum_{k \in I_c} \left|X_{mnk}, \overline{0}\right| = 0.$$

In this case write $X_{mnk} \to^{[V,\lambda]} \overline{0}$.

Definition 2.5 *A triple sequence spaces of* (X_{mnk}) *is said to be* $\lambda-$ *statistically convergent (or shortly:* $S_\lambda -$ *convergent) to* $\overline{0}$ *if for any* $\varepsilon > 0$,

$$\lim_{abc \to \infty} \frac{1}{\lambda_{abc}} \left|\left\{(m, nk) \in I_{abc} : \left|X_{mnk}, \overline{0}\right| \geq \varepsilon\right\}\right| = 0.$$

In this case we write $S_\lambda - \lim X_{mnk} = \overline{0}$ or by $X_{mnk} \to^{S_\lambda} \overline{0}$.

Let f be a Musielak Orlicz function; q be positive real number then we define the following definitions:

Let (γ, η) as a sliding window pair and $g : [0, \infty) \to \mathbb{R}^3$ a measurable function. Then;

Definition 2.6 *The function* g *is* $N(\gamma,\eta,f,q)$ *summable to* $\overline{0}$ *and write* $N(\gamma,\eta,f,q)-\lim g=\overline{0}$ $($*or* $g\to\overline{0}$ $N(\gamma,\eta,f,q))$ *if and only if*

$$\lim_{abc\to\infty}\frac{1}{\mu(I_{abc})}\int_{I_{abc}}f\left(\left|g(t),\overline{o}\right|^{q}\right)dt=0.$$

Definition 2.7 *Let* α,q *be non negative real number. The function* g *is* $N(\gamma,\eta,f,q)$ *summable to 0. A triple sequence spaces of* (X_{mnk}) *of random variables is said to be rough* $[V,\lambda]-$ *summable in probability to* $X:W\times W\times W\to\mathrm{R}\times\mathrm{R}\times\mathrm{R}$ *with respect to the roughness of degree* α *(or shortly:* $\alpha-[V,\lambda]-$ *summable in probability to 0 if for any* $\varepsilon>0$,

$$\lim_{abc\to\infty}\frac{1}{\lambda_{abc}}\frac{1}{\mu(I_{abc})}\mu\left(t\in I_{abc}:f\left(P\left(\left|X_{mnk}(g(t)),\overline{0}\right|^{q}\right)\geq\alpha+\varepsilon\right)\right)=0.$$

In this case we write $X_{mnk}\to_{\alpha}^{[V,\lambda]^{P}}\overline{0}$. The class of all rough $[V,\lambda]-$ summable triple sequence spaces of random variables in probability will be denoted simply by $\alpha[V,\lambda]^{P}$.

Definition 2.8 *Let* α,q *be non negative real number. The function* g *is* $N(\gamma,\eta,f,q)$ *summable to 0. A triple sequence spaces of* (X_{mnk}) *of random variables is said to be rough* $\lambda-$ *statistically convergent in probability to* $X:W\times W\times W\to\mathrm{R}\times\mathrm{R}\times\mathrm{R}$ *with respect to the roughness of degree* α *(or shortly:* $\alpha-\lambda-$ *statistically convergent in probability to* $\overline{0}$*) if for any* $\varepsilon,\delta>0$,

$$\lim_{abc\to\infty}\frac{1}{\lambda_{abc}}\frac{1}{\mu(I_{abc})}\mu\left(|\{t\in I_{abc}:f(P\left(\left|X_{mnk}(g(t)),\overline{0}\right|\geq\alpha+\varepsilon\right)\geq\delta\}|\right)=0.$$

In this case we write $S(\gamma,\eta,f,q)X_{mnk}\to_{\alpha}^{S_{\lambda}^{P}}\overline{0}$. The class of all $\alpha-\lambda-$ statistically convergent triple sequence spaces of random variables in probability will be denoted simply by αS_{λ}^{P}.

Remark 2.1 *Let* f *be an sliding window pair of measurable Musielak-Orlicz function of triple sequence is*

$$\left\|\chi_{f}^{3},(d(x_{1}),d(x_{2}),\cdots,d(x_{n-1}))\right\|_{p}$$
$$=\left[f_{mnk}\left(\left\|\mu_{mnk}(X(g(t))),(d(x_{1}),d(x_{2}),\cdots,d(x_{n-1}))\right\|_{p}\right)\right],$$

where $\mu_{mnk}(X(g(t)))=\left(((m+n+k)!X_{mnk}(g(t)))^{1/m+n+k},\overline{0}\right)$.

3 Main Results

Theorem 3.1 *Let a triple sequence spaces of* $(X_{mnk}(g(t)))$ *of random variables of sliding window pair of measurable Musielak Orlicz functions are equivalent:*

(i) $\left\| \chi_f^3 \left(X_{mnk}\left(g\left(t\right)\right)\right), \left(d\left(x_1\right), d\left(x_2\right), \cdots, d\left(x_{n-1}\right)\right) \right\|_p$ is $\alpha - [V, \lambda] -$ summable in probability to $\overline{0}$.

(ii) $\left\| \chi_f^3, \left(d\left(x_1\right), d\left(x_2\right), \cdots, d\left(x_{n-1}\right)\right) \right\|_p$ is $\alpha - \lambda -$ statistically convergent in probability to $\overline{0}$.

Proof. Similar to the proof of Theorem (3.1) in (see [17]).

Theorem 3.2 *Let a triple sequence spaces of* $\left(X_{mnk}\left(g\left(t\right)\right)\right)$ *of random variables of sliding window pair of measurable Musielak Orlicz functions.*

If

$$\left\| \chi_f^3 \left(X_{mnk}\left(g\left(t\right)\right)\right), \left(d\left(x_1\right), d\left(x_2\right), \cdots, d\left(x_{n-1}\right)\right) \right\|_p \to_{\alpha}^{S^P} \overline{0}$$

and

$$\left\| \chi_f^3 \left(Y_{mnk}\left(g\left(t\right)\right)\right), \left(d\left(x_1\right), d\left(x_2\right), \cdots, d\left(x_{n-1}\right)\right) \right\|_p \to_{\beta}^{S^P} \overline{0}$$

then

$$\lim_{abc\to\infty} \frac{1}{\lambda_{abc}} \frac{1}{\mu\left(I_{abc}\right)}$$

$$\mu\left\{ P\left(\left| \left[f_{mnk}\left(\left\| \mu_{mnk}\left(X\left(g\left(t\right)\right)\right), \left(d\left(x_1\right), d\left(x_2\right), \cdots, d\left(x_{n-1}\right)\right) \right\|_p \right) \right] \right| \geq \alpha + \varepsilon \right) \right\} = 0.$$

Proof. Similar to the proof of Theorem (3.1) in (see [16]).

Theorem 3.3 *Let a triple sequence spaces of* $\left(X_{mnk}\left(g\left(t\right)\right)\right)$ *of random variables of sliding window pair of measurable Musielak Orlicz functions. If* $\lambda \in \Im$ *is such that* $\frac{\lambda_{abc}\mu\left(I_{abc}\right)}{(abc)} = 1$ *then* $\alpha S_\lambda^P \subset \alpha S^P$.

Proof. Let $0 < \eta < 1$ be given. Since $\lim_{abc\to\infty} \frac{\lambda_{abc}\mu\left(I_{abc}\right)}{(abc)} = 1$, we can choose $(u, v, w) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N}$ such that $\left| \frac{\lambda_{abc}\mu\left(I_{abc}\right)}{(abc)} - 1 \right| < \frac{\eta}{2}$ for all $(a, b, c) > (u, v, w)$. Now, for $\varepsilon, \delta > 0$

$$\frac{1}{abc} \left| \left\{ (mnk) \leq (abc) : \mu\left(P\left(\left| \left[f_{mnk}\left(\left\| \mu_{mnk}\left(X\left(g\left(t\right)\right)\right), \left(d\left(x_1\right), d\left(x_2\right), \cdots, d\left(x_{n-1}\right)\right) \right\|_p \right) \right] \right| \right.\right.\right.\right.$$

$$\geq \alpha + \varepsilon) \geq \delta)\}\Big|$$

$$= \frac{1}{abc}\Big|\big((mnk) \leq (abc) - \lambda_{abc}\mu(I_{abc}) :$$
$$\mu\left\{P\left(\left\|\left[f_{mnk}\left(\left\|\mu_{mnk}\left(X\left(g\left(t\right)\right)\right),\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\right\|_p\right)\right]\right\| \geq \alpha + \varepsilon\right) \geq \delta\right\}\Big)\Big|$$

$$= \frac{1}{abc}\Big|\big((mnk) \in I_{abc} :\big)\Big|$$
$$\mu\left\{P\left(\left\|\left[f_{mnk}\left(\left\|\mu_{mnk}\left(X\left(g\left(t\right)\right)\right),\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\right\|_p\right)\right]\right\| \geq \alpha + \varepsilon\right) \geq \delta\right\}\Big)\Big|$$

$$\leq \frac{(abc) - \lambda_{abc}\mu(I_{abc})}{(abc)} + \frac{1}{abc}\Big|\big((mnk) \in I_{abc} :\big)\Big|$$
$$\mu\left\{P\left(\left\|\left[f_{mnk}\left(\left\|\mu_{mnk}\left(X\left(g\left(t\right)\right)\right),\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\right\|_p\right)\right]\right\| \geq \alpha + \varepsilon\right) \geq \delta\right\}\Big)\Big|$$

$$\leq 1 - \left(1 - \frac{\eta}{2}\right) + \frac{1}{abc}\Big|\big((mnk) \in I_{abc} :\Big|$$
$$\mu\left\{P\left(\left\|\left[f_{mnk}\left(\left\|\mu_{mnk}\left(X\left(g\left(t\right)\right)\right),\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\right\|_p\right)\right]\right\| \geq \alpha + \varepsilon\right) \geq \delta\right\}\Big)\Big|,$$

then

$$= \frac{\eta}{2} + \frac{\lambda_{abc}\mu(I_{abc})}{abc}\frac{1}{\lambda_{abc}\mu(I_{abc})}\Big|\big((mnk) \in I_{abc} :\Big|$$
$$\mu\left\{P\left(\left\|\left[f_{mnk}\left(\left\|\mu_{mnk}\left(X\left(g\left(t\right)\right)\right),\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\right\|_p\right)\right]\right\| \geq \alpha + \varepsilon\right) \geq \delta\right\}\Big)\Big|$$

$$< \frac{\eta}{2} + \frac{1}{\lambda_{abc}\mu(I_{abc})}\Big|\big((mnk) \in I_{abc} :\Big|$$
$$\mu\left\{P\left(\left\|\left[f_{mnk}\left(\left\|\mu_{mnk}\left(X\left(g\left(t\right)\right)\right),\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\right\|_p\right)\right]\right\| \geq \alpha + \varepsilon\right) \geq \delta\right\}\Big)\Big|$$

holds for all $(a,b,c) \geq (u,v,w)$.

Theorem 3.4 *Let a triple sequence spaces of* $\left(X_{mnk}\left(g\left(t\right)\right)\right)$ *of random variables of sliding window pair of measurable Musielak Orlicz functions,* $\alpha S^P \subset \alpha S_\lambda^P$ *if and only if* $\lim_{abc\to\infty}\frac{\lambda_{abc}}{(abc)} > 0.$

Proof. Let $\lim_{abc\to\infty}\frac{\lambda_{abc}\mu\left(I_{abc}\right)}{(abc)}>0.$ Then for $\varepsilon,\delta>0,$ we have

$$\frac{1}{abc}\left|\left((mnk)\le(abc):\right.\right.$$
$$\left.\mu(P\left(\left|\left[f_{mnk}\left(\left\|\mu_{mnk}\left(X\left(g\left(t\right)\right)\right),\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\right\|_p\right)\right]\right|\ge\alpha+\varepsilon\right)\ge\delta\}\right)\right|$$
$$\ge\frac{1}{abc}\left|\left((mnk)\in I_{abc}:\right.\right.$$
$$\left.\mu\left\{P\left(\left|\left[f_{mnk}\left(\left\|\mu_{mnk}\left(X\left(g\left(t\right)\right)\right),\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\right\|_p\right)\right]\right|\ge\alpha+\varepsilon\right)\ge\delta\right\}\right|$$
$$=\frac{\lambda_{abc}\mu\left(I_{abc}\right)}{abc}$$
$$\times\frac{1}{\lambda_{abc}\mu\left(I_{abc}\right)}\left|\left((mnk)\in I_{abc}:\right.\right.$$
$$\left.\mu\left\{P\left(\left|\left[f_{mnk}\left(\left\|\mu_{mnk}\left(X\left(g\left(t\right)\right)\right),\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\right\|_p\right)\right]\right|\ge\alpha+\varepsilon\right)\ge\delta\right\}\right)\right|$$

Taking limit $abc\to\infty$ we get

$$\left\|\chi_f^3\left(X_{mnk}\left(g\left(t\right)\right)\right),\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\right\|_p\to_{\alpha}^{S^P}\bar{0}\Rightarrow$$

$$\left\|\chi_f^3\left(X_{mnk}\left(g\left(t\right)\right)\right),\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\right\|_p\to_{\alpha}^{S_\lambda^P}\bar{0}.$$

Conversely, let $\lim_{abc\to\infty}\frac{\lambda_{abc}\mu\left(I_{abc}\right)}{(abc)}=0$ then we can choose a subsequence $\left(a_u,b_v,c_w\right)_{u,v,w\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}}$ such that $\frac{\lambda_{a_u,b_v,c_w}\mu\left(I_{a_ub_vc_w}\right)}{a_u,b_v,c_w}<\frac{1}{uvw}$ for all $u,v,w\in\mathbb{N}^3$. Define a sliding window rough triple sequence spaces of $\left(X_{mnk}\left(g\left(t\right)\right)\right)$ for measurable function of random variables whose probability density function is

$$\mu_{abc}\left(X\left(g\left(t\right)\right)\right)=\begin{cases}1, & if\ 0<X<1\\ 0, & otherwise\end{cases},\text{where}(abc)\in I_{abc}\text{forsome}(uvw)\in\mathbb{N}^3$$

Let $0<\varepsilon,\delta<1.$ Then

$$\mu\left(P\left(\left|\left[f_{mnk}\left(\left\|\mu_{mnk}\left(X\left(g\left(t\right)\right)\right),\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\right\|_p\right)\right]\right|\ge1+\varepsilon\right)\right)$$
$$=\begin{cases}1, & if\ (abc)\in I_{abc}\text{forsome}(uvw)\in\mathbb{N}^3\\ \left(1-\frac{\varepsilon}{2}\right)^n, & otherwise.\end{cases}$$

We have

$$\frac{1}{\lambda_{abc}\mu(I_{abc})}\left|\left((mnk)\in I_{abc}:\right|\right.$$

$$\left.\mu\left\{P\left(\left\|\left[f_{mnk}\left(\left\|\mu_{mnk}\left(X\left(g\left(t\right)\right)\right),\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\right\|_p\right)\right]\right\|\geq 1+\varepsilon\right)\geq\delta\right\}\right)\right|$$

$$=\begin{cases}1, & if\ (abc)\in I_{abc}\text{forsome}(uvw)\in \mathbb{N}^3\\ 0, & otherwise\ .\end{cases}$$

Hence

$$\left\|\chi_f^3\left(X_{mnk}\left(g\left(t\right)\right)\right),\left(d\left(x_1\right),d\left(x_2\right),\cdots,d\left(x_{n-1}\right)\right)\right\|_p \notin_{\alpha}^{S_\lambda^P}\ .$$

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