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Research Article

Proximal Mapping Involving Fixed Points on closed Ball and Application to Integral Equation

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Abstract: The aim of this paper is to introduce semi $\alpha*$-admissible proximal multivalued mapping and establish fixed point results satisfying $\alpha*$-ψ-contractive conditions in left K-sequentially complete dislocated quasi metric space. We derive some new fixed point theorems under the improve approach of ordered metric space. An example has been constructed to demonstrate the novelty of these results. As an application, we derive some new fixed point theorems for graphic contractions and obtain existence and uniqueness of solutions of integral equations.

Keywords: Fixed point, left K-sequentially complete dislocated quasi metric space, closed ball, semi $\alpha*$-admissible.

INTRODUCTION

Let S: X→X be a mapping. A point x ∈ X is called a fixed point of S if x = Sx. Many results appeared in literature related to the fixed point of mappings which are contractive on the whole domain. It is possible that S: X→X is not a contraction but S: Y→X is a contraction, where Y is a closed ball in X. One can obtain fixed point results for such mapping by using suitable conditions. Recently, Hussain et al. [20] proved a result concerning the existence of fixed points of a mapping satisfying a contractive conditions on closed ball (see also [5, 6, 7, 8, 36, 37, 38]).

The notion of dislocated topologies have useful applications in the context of logic programming semantics (see [16]). Dislocated metric space (metric-like space) (see [4, 24, 26, 31]) is a generalization of partial metric space (see [3, 23, 29, 33]).

Furthermore, dislocated quasi metric space (quasi-metric-like space) (see [35, 40, 41]) generalized the idea of dislocated metric space and quasi-partial metric space (see [15, 25, 36]). A complete dislocated quasi metric space is also a generalization of 0-complete quasi-partial metric space (see [32, 36]).

Definition 1.1. [40] Let X be a nonempty set and let d_q : X ×X → [0,∞) be a function, called a dislocated quasi metric (or simply d_q-metric) if the following conditions hold for any x,y,z ∈ X.

(i) If d_q(x, y) = d_q (y, x) = 0, then x = y.

(ii) (ii) d_q (x, y)≤ d_q (x, z) + d_q (z, y).

The pair (X, d_q) is called a dislocated quasi metric space.

It is clear that if d_qx, y) = q (y, x) = 0, then from (i), x = y. But if x = y, d_q (x, y) may not be 0. It is observed that if d_q (x, y) = d_q (y, x) for all x,y ∈ X then (X, d_q) becomes a dislocated metric space (metric-like space). We will denote (X, d_l) a dislocated metric space. For x ∈ X and

"ε> 0, $\overline{B(x,\varepsilon)} = \{y \in X: d_q(x,y) \leq \varepsilon\}$ is a closed ball in (X, dq).

Example 1.2. If X = R+ ∪ {0} then dq(x, y) = x + max{x, y} defines a dislocated quasi metric dq on X.

Reilly et al. [30] introduced the notion of left (right) K-Cauchy sequence and left (right) K-sequentially complete spaces (see also [14]). We use this concept to establish the following definition.

Definition 1.4. Let K be a nonempty subset of dislocated quasi metric space

X and let x ∈ X. An element yy_0 ∈ K is called a best approximation in K if d_q (x, K) = d_q (x, y_0), where d_q (x, K) = $inf_{y \in K} d_q$ (x, y)

If each x ∈ X has at least one best approximation in K, then K is called a proximal set.

We denote CP(X) be the set of all closed proximal subsets of X.

Let Ψ denote the family of all nondecreasing functions ψ: [0, +∞) →[0, +∞) such that $\sum_{n=1}^{+\infty} \psi^n(t) < +\infty$ for all t> 0, where ψ^n is the nth iterate of : If ψ∈ Ψ, then ψ(t)<t for all t> 0.

Definition 1.5. Let (X, d) be a metric space, S : X→CP(X) be a multivalued mapping and α: X× X→ [0;+∞). Let A⊆X; we say that S is semi α∗-admissible on A, whenever α(x, y)≥1 implies that α(Sx, Sy)≥1, for all x,y∈A, where α(Sx, Sy) = inf {α(a, b) : a∈Sx,b∈ Sy}. If A = X, then we say that S is α∗-admissible on X.

Definition 1.6. The function H_{dq} : CP(X) ×CP(X) → X, defined by

H_{dq} (A, B)= $max\{sup_{a\in A}\, dq(a,B)\ sup_{b\in B} dq(A,b)\}$

is called dislocated quasi hausdorff metric on CP(X).

Let X be a nonempty set. Then $(X, \leq, d_l)$ is called a preordered dislocated metric space if d_l is a dislocated metric on X and $\leq$ is a preorder on X. Let $(X, \leq, d_q)$ be a preordered metric space and A,B⊆X. We say that A≤ B whenever for each a ∈ A there exists b∈ B such that a≤ b. Also, we say that A ≤r B whenever for each a ∈ A and b∈ B we have a≤ b.

2 RESULTS AND DISCUSSION

Let (X, d_q) be a dislocated quasi metric space, $x_0 \in X$ and $S: X \to CP(X)$ be a multivalued mapping on X. Then there exist $x_1 \in Sx_0$ such that $d_q(x_0, Sx_0) = d_q(x_0, x_1)$. Let $x_2 \in Sx_1$ be such that $d_q(x_1, Sx_1) = d_q(x_1, x_2)$. Continuing this process, we construct a sequence x_n of points in X such that $x_{n+1} \in Sx_n$ and $d_q(x_n, Sx_n) = d_q(x_n, x_{n+1})$. We denote this iterative sequence by $\{XS(x_0)\}$.

Theorem 2.1. Let (X, d_q) be a left K-sequentially complete dislocated quasi metric space, $r > 0$, $x_0 \in \overline{B_{dq}(x_0, r)}$ and $S : X \to CP(X)$ be a semi α∗-admissible multifunction on $\overline{B_{dq}(x_0, r)}$. Assume that for $\psi \in \Psi$ such that

$$\alpha *(Sx, Sy)H_{dq}(Sx, Sy) \leq \psi(M_q(x, y)) \text{ for all } x, y \in \overline{B_{dq}(x_0, r)}$$

(2.1)

where

$$M_q(x,y) = \max\{d_q(x,y), d_q(x,Tx), d_q(y,Ty)\}$$

and $$\sum_{i=0}^{n} \psi^i(d_q(x_0, Sx_0)) \leq r$$ for all $n \in \mathbb{N} \cup \{0\}$. (2.2)

If $\{XS(x_0)\}$ is a sequence in $\overline{B_{dq}(x_0, r)}$ and $\{XS(x_0)\} \to x$ and $\alpha(x_n, x_{n+1}) \geq 1$ for $x_n, x_{n+1} \in \{XS(x_0)\}$, $n \in \mathbb{N} \cup \{0\}$. then $\alpha(x_n, x) \geq 1$ or $\alpha(x, x_n) \geq 1$ for all $n \in \mathbb{N} \cup \{0\}$.. Also there

exist $x_1 \in Sx_0$ such that $\alpha(x_0, x_1) \geq 1$. Then S has a fixed point in $\overline{B_{dq}(x_0, r)}$.

Proof. As $x_0 \in \overline{B_{dq}(x_0, r)}$ and $S : X \to CP(X)$ be a multivalued mapping on X, then there

exist $x_1 \in Sx_0$ such that $d_q(x_0, Sx_0) = d_q(x_0, x_1)$. If $x_0 = x_1$, then x_0 is a fixed point in $\overline{B_{dq}(x_0, r)}$ of S. Let $x_0 \neq x_1$. From (2.2), we get

$$d_q(x_0, x_1) \leq \sum_{i=0}^{n} \psi^i(d_q(x_0, x_1)) \leq r.$$

It follows that, $x_1 \in \overline{B_{dq}(x_0, r)}$

Since $\alpha(x_0, x_1) \geq 1$ and S is semi $\alpha*$-admissible multifunction on $\overline{B_{dq}(x_0, r)}$, so $\alpha*(Sx_0; Sx_1) \geq 1$. Also there exist $x_2 \in Sx_1$ such that $d_q(x_1, Sx_1) = d_q(x_1, x_2)$, If $x_1 = x_2$, then x_1 is a fixed point of S in $\overline{B_{dq}(x_0,r)}$. Let $x_1 \neq x_2$. Now,

$$d_q(x_1, x_2)= d_q(x_1, Sx_1)\ H_{dq}(Sx_0, Sx_1)$$

$$\leq \alpha *(Sx_0, Sx_1)H_{dq}(Sx_0, Sx_1)$$

Note that $x_2 \in \overline{B_{dq}(x_0,r)}$ because

$d_q(x_0, x_2)\leq d_q(x_0, x_1) + d_q(x_1,$

$x_2)$

$$\leq d_q(x_0;x_1) + \alpha * (Sx_0, Sx_1)H_{dq}(Sx_0, Sx_1)$$

$$\leq d_q(x_0, x_1) + \psi(M_q(x_0;x_1)) \text{ by (2.1)}$$

$$= d_q(x_0;x_1) + \psi(\max\{d_q(x_0, x_1),d_q(x_0, Sx_0),d_q(x_1,Sx_1)\})$$

$$\leq d_q(x_0;x_1) + \psi(\max\{d_q(x_0, x_1),d_q(x_1, Sx_1)\}).$$

If $\max\{d_q(x_0, x_1),d_q(x_1, Sx_1)\}.= d_q(x_1, Sx_1)$, then $d_q(x_1, Sx_1)\leq\psi\ (d_q(x_1, Sx_1))$. Since $(t) < t$ for all $t> 0$: Then we get a contradiction. Hence, we obtain

$\max\{d_q(x_0, x_1),d_q(x_1, Sx_1)\}.= d_q(x_0, x_1)$. So

$$\mathrm{d}(x_1, Tx_1)\leq d_q(x_0, x_1)+\psi\ (d_q(x_0, x_1))$$

$$\leq \sum_{i=0}^{n} \psi^i \left(dq(x0,x1)\right) \leq \mathrm{r}.$$

As $\alpha * (Sx_0; Sx_1) \geq 1$, $x_1 \in Sx_0$ and $x_2 \in Sx_1$ so $\alpha(x_1, x_2) \geq 1$. As S is semi $\alpha *$-admissible multifunction on thus $\alpha * (Sx_1, Sx_2) \geq 1$. Let

$x_2, \ldots, x_j \in \overline{B_{dq}(x_0, r)}$ for some $j \in \mathrm{N}$ such that $x_{j+1} \in Sx_j$ and $d_q(x_j, Sx_j) = d_q(x_j, x_{j+1})$. As $\alpha * (Sx_0, Sx_1) \geq 1$, we have $\alpha\,(x_2, x_3) \geq 1$; which further implies

$\alpha * (Sx_2, Sx_3) \geq 1$. Continuing this process, we have $\alpha * (Sx_{j\ 1}, Sx_j)\ 1$.

Now, $x_j \in Sx_j$, $x_{j+1} \in Sx_j$, we have $dq(xj, xj{+}1) = dq(xj, Sxj) \leq Hdq(Sxj\ 1, Sxj)$

$$\leq \alpha * (Sxj\ 1, Sxj) Hdq(Sxj\ 1, Sxj)$$

$$\leq \psi\,(M_q(x_{j\ 1}, x_j))$$

$$\leq (\ \max\{dq(xj\ 1, xj); dq(xj\ 1, Sxj\ 1); dq(xj, Sxj)\})$$

$$\leq (\ \max\{dq(xj\ 1, xj), dq(xj, Sxj)\})$$

If $\max\{dq(xj\ 1, xj), dq(xj, Sxj)\} = dq(xj, Sxj)$, then $dq(xj, Sxj) \leq \psi(dq(xj, Sxj))$. Since $(t) < t$ for all $t > 0$. Then we get a contradiction. Hence, we obtain $\max\{d_q(x_{j\ 1}, x_j), d_q(x_{j\ 1}, Sx_{j\ 1})\} = d_q(x_{j\ 1}, x_j)$,

$$d_q(x_j, x_{j+1}) \leq \psi^j(d_q(x_0, x_1)) \tag{2.3}$$

$$\begin{aligned} d_q(x_0, x_{j+1}) &\leq d_q(x_0, x_1) + \ldots + d_q(x_j, x_{j+1}) \\ &\leq d_q(x_0, x_1) + \ldots + \psi^j(d_q(x_0, x_1)) \\ &= \sum_{i=0}^{j} \psi^i(d_q(x_0, x_1)) \leq r. \end{aligned}$$

Thus $x_{j+1} \in \overline{B_{dq}(x_0, r)}$. As $\alpha * (Sx_{j\ 1}; Sx_j) \geq 1$, $x_j \in Sx_{j-1}$, $x_{j+1} \in Sx_j$, we have $\alpha(x_j; x_{j+1}) \geq 1$.

Also S is semi $\alpha *$ -admissible multifunction on $\overline{B_{dq}(x_0, r)}$, therefore $\alpha * (Sx_j, Sx_{j+1}) \geq 1$. Hence by mathematical induction, $x_n \in \overline{B_{dq}(x_0, r)}$ and $\alpha * (Sx_n, Sx_{n+1}) \geq 1$ for all $n \in$ N. Now inequality (2.3) can be written as

$$d_q(x_n, x_{n+1}) \leq \psi^n(d_q(x_0, x_1)) \text{ for all } n \in \text{N}.$$

Fix $\varepsilon > 0$ and let $n(\varepsilon) \in$ N such that $\sum_{i=0}^{n} \psi^i \left(dq(x0, x1)\right) < \varepsilon$. Let $n, m \in$ N with $m > n > k(\varepsilon)$, then, we obtain,

$$dq(xn, xm) \leq \sum_{k=n}^{m-1} dq(xk, xk+1) \leq \sum_{k=n}^{m-1} \psi^k \left(dq(x0, x1)\right) \leq \sum_{n \geq k(\varepsilon)} \psi k(dq(x0; x1)).$$

Thus we proved that $\{x_n\}$ is a Cauchy sequence in $(\overline{B_{dq}(x_0, r)}, d_q)$. As every closed ball in a left K-sequentially complete dislocated quasi metric space is left K-sequentially complete, so there exists $x^* \in \overline{B_{dq}(x_0, r)}$ such that $x_n \to x^*$, and

$$lim_{n \to \infty}\ d_q(x_n, x) = lim_{n \to \infty}\ d_q(x, x_n)$$

Note that $\{x_n\}$ is a $\{XS(x_0)\}$ in $\overline{B(d_q(x_0,r))}$. As $\alpha*(Sx_n, Sx_{n+1})\geq 1$ for all $n\in$ N, we have $\alpha(xn+1;xn+2)\geq 1$ for all $n\in$ N$\cup\{0\}$. By assumption, we have $\alpha(x_n, x)\geq 1$ for all $n\in$ N$\cup\{0\}$. Thus $\alpha*(Sx_n, Sx)\geq 1$. Now

$d_q(x, Sx)\leq d_q(x, xn+1) + dq(xn+1, Sx)$

$\leq d_q(x, x_{n+1}) + H_{dq}(Sx_n, Sx)$

$\leq d_q(x, x_{n+1}) + \alpha*(Sx_n, Sx)H_{dq}(Sx_n, Sx)$

$\leq d_q(x, x_{n+1}) + \psi(\max\{d_q(x_n, x), d_q(x_n, x_{n+1}), d_q(x, Sx)\})$:

Letting $n\to\infty$, we obtain $d_q(x;Sx) = 0$: Similarly, if $\alpha(x, x_n)\geq 1$ for all $n\in$ N$\cup\{0\}$.

Thus $\alpha*(Sx, Sx_n)\geq 1$. Now

$$d_q(Sx, x)\leq \psi(d_q(x, x_n)) + d_q(x_{n+1}, x).$$

We obtain $d_q(Sx, x) = 0$. Hence $x\in Sx$: So S has a fixed point in $\overline{B(d_q(x_0,r))}$. Corollary 2.2. Let (X, d_q) be a left K-sequentially complete dislocated quasi metric space, $r > 0$, $x_0\in\overline{B_{dq}(x_0,r)}$ and $S : X\to CP(X)$ be a semi $\alpha*$-admissible multifunction on $\overline{B_{dq}(x_0,r)}$. Assume that for $\psi\in\Psi$ such

$\alpha *(Sx, Sy)H_{dq}(Sx, Sy) \leq \psi(M_q(x, y))$ for all $x, y \in \overline{B_{dq}(x_0, r)}$

(2.1)

where

$$M_q(x,y) = \max\{d_q(x,y), d_q(x,Tx), d_q(y,Ty)\}$$

and $\sum_{i=0}^{n} \psi^i \left(dq(x0, x1)\right) \leq$ r. for all $n \in$ N∪ {0}.

If $\{XS(x_0)\}$ is a sequence in $\overline{B_{dq}(x_0, r)}$ and $\{XS(x_0)\} \rightarrow x$ and α $(x_n, x_{n+1}) \geq 1$ for x_n, $x_{n+1} \in \{XS(x_0)\}$, $n \in$ N∪ {0}. then $\alpha(x_n, x) \geq 1$ or $\alpha(x, x_n) \geq 1$ for all $n \in$ N∪ {0}.. Also there

exist $x_1 \in Sx_0$ such that $\alpha(x_0, x_1) \geq 1$. Then S has a fixed point in $\overline{B_{dq}(x_0, r)}$.

Corollary 2.3. Let $(X, \leq, d_l)$ be a preordered left K-sequentially complete dislocated quasi metric space, $S : X \rightarrow CP(X)$. Suppose there exists $\psi \in \Psi$, with

$H_{dq}(Sx, Sy) \leq \psi(d_q(x;y))$, for all elements x, y in $\overline{B_{dq}(x_0, r)}$.with x y,

and $\sum_{i=0}^{n} \psi^i \left(dq(x_0, Sx_0)\right) \leq$ r, for all $n \in$ N∪ {0}.

for $x_0 \in \overline{B_{dq}(x_0, r)}$, $n \in$ N, $r > 0$. If $\{XS(x_0)\}$ is a sequence in $\overline{B_{dq}(x_0, r)}$ and $\{XS(x_0)\} \rightarrow x$ and $x_n \leq x_{n+1}$ for x_n, $x_{n+1} \in \{XS(x_0)\}$, then $x \leq x_n$ or $x_n \leq x$, for all $n \in$ N∪ {0}. Also there exist $x_1 \in Sx_0$ such that $x_0 \leq x_1$. If x, $y \in$

$\overline{B_{dq}(x_0, r)}$ such that $x \leq y$ implies $Sx \leq_r Sy$. Then there exists a point x in $\overline{B_{dq}(x_0, r)}$ such that $x \in Sx$.

Corollary 2.4. Let $(X, \leq, d_l)$ be a preordered left K-sequentially complete dislocated quasi metric space, $S : X \to CP(X)$. Suppose there exists $k \in [0,1)$ with

$$H_{dq}(Sx, Sy) \leq kd_q(x, y); \text{ for all elements } x, y \text{ in } \overline{B_{dq}(x_0, r)} \text{ with } x \leq y$$

$$\text{and } \sum_{i=0}^{n} k^i d_q(x_0, Sx_0) \leq r \text{ for all } n \in N \cup \{0\}.$$

$i=0$

for $x_0 \in \overline{B_{dq}(x_0, r)}$, $n \in N$, $r > 0$. If $\{XS(x_0)\}$ is a sequence in $\overline{B_{dq}(x_0, r)}$ and $\{XS(x_0)\} \to x$ and $x_n \leq x_{n+1}$ for $x_n, x_{n+1} \in \{XS(x_0)\}$, then $x \leq x_n$ or $x_n \leq x$, for all $n \in N \cup \{0\}$. Also there exist $x_1 \in Sx_0$ such that $x_0 \leq x_1$. If $x, y \in \overline{B_{dq}(x_0, r)}$ such that $x \leq y$ implies $Sx \leq_r Sy$. Then there exists a point x in $\overline{B_{dq}(x_0, r)}$ such that $x \in Sx$.

Let $f : X \to X$ be a self-mapping of a set X and $\alpha: X \times X \to [0, 1)$ be a mapping, then the mapping f is called semi α -admissible if, $A \subseteq X$, $x,y \in A$, $\alpha(x,y) \geq 1$ implies $\alpha(fx, fy) \geq 1$. If $A = X$, then the mapping f is called α -admissible.

Corollary 2.5. Let (X, d_q) be a left K-sequentially complete dislocated quasi metric space and $S : X \to X$, $r > 0$ and x_0 be an arbitrary point in $\overline{B_{dq}(x_0, r)}$. Suppose there exists, $\alpha : X \times X \to [0, 1)$ be a semi α -admissible mapping on $\overline{B_{dq}(x_0, r)}$

For $\psi \in \Psi$, assume that,

$$x, y \in \overline{B_{dq}(x_0, r)}\ \alpha(x;y) \geq 1 \mapsto d_q(Sx, Sy) \leq (d_q(x, y))$$

$$\text{and } \sum_{i=0}^{n} \psi^i \left(dq(x_0, Sx_0)\right) \leq r, \text{ for all } n \in \mathrm{N} \cup \{0\}.$$

Suppose that the following assertions hold:

(i) $\alpha(x_0; Sx_0) \geq 1$;

(ii) for a Picard sequence $x_{n+1} = Sx_n$ in $\overline{B_{dq}(x_0, r)}$ such that $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathrm{N} \cup \{0\}$. and $x_n \to u \in \overline{B(x_0, r)}$ as $n \to \infty$ then $\alpha(x_n, u) \geq 1$ for all $n \in \mathrm{N} \cup \{0\}$.

Then, there exists a point x in $\overline{B_{dq}(x_0, r)}$ such that $x = Sx$.

Recall that if $(X; \leq)$ is a preordered set and $T : X \to X$ is such that for $x, y \in X$; with $x \leq y$ implies $Tx \leq Ty$, then the mapping T is said to be non-decreasing.

Corollary 2.6. Let $(\mathrm{X}, \leq, d_q)$ be a preordered left K-sequentially complete dislocated quasi metric space and let $S : X \to X$ be nondecreasing mapping and $x_0 \in \overline{B(x_0, r)}$. Suppose that the following assertions hold:

(i) there exists $k \in [0,1)$ such that $d_q(Sx, Sy) \leq k d_q(x, y)$ for all $x, y \in \overline{B_{dq}(x_0, r)}$ with $x \leq y$;

(ii) $x_0 \leq Sx_0$ and $\sum_{i=0}^{n} k^i d_q(x_0, Sx_0) \leq r$ for all $n \in \mathrm{N} \cup \{0\}$.

(iii) for a Picard sequence $x_{n+1} = Sx_n$ in $\overline{B_{dq}(x_0, r)}$ such that $x_n \leq x_{n+1}$ for all $n \in$ N∪ {0}. and $x_n \to u \in \overline{B(x_0, r)}$ as $n \to \infty$ then $x_n \leq u$ for all $n \in$ N∪ {0}.

Then, there exists a point x in $\overline{B_{dq}(x_0, r)}$ such that $x = Sx$.

Then S has a fixed point.

Example 2.7. Let $X = Q^+ \cup \{0\}$ and let $d_q : X \times X \to X$ be the left K-sequentially complete dislocated quasi metric on X defined by,

$$d_q(x, y) = \frac{x}{2} + y \quad \text{for all } x, y \in X.$$

Define the multivalued mapping $S : X \to CP(X)$ by

$$Sx = \begin{cases} [\frac{1}{2}, \frac{2}{3}x] \text{ if } x \in [0, 1] \\ [x, x+1] \text{ if } x \in (1, \infty) \end{cases}$$

Considering, $x_0 = 1$, $r = 4$, then $\overline{B_{dq}(x_0, r)} = [0, 1] \setminus X$. Now $d_q(x_0, Sx_0)$ $= d_q(1, S1) = d_q(1, \frac{1}{2}) = 1$. Let $\psi(t) = \frac{t}{3}$ and

$$\alpha(x, y) = \begin{cases} 1 \text{ if } x, y \in [0, 1] \\ \frac{3}{2} \text{ Otherwise.} \end{cases}$$

Now

$$\alpha *(S2, S4) H_{d_q}(S2, S4) = (\frac{3}{2})6 > \psi(M_q(x, y))$$

So the contractive condition does not hold on X. Clearly

$$\alpha *(Sx, Sy) H_{dq}(Sx, Sy) \leq (M_q(x, y)) \text{ for all } x, y \in \overline{B_{dq}(x_0, r)}.$$

So the contractive condition holds on $\overline{B_{dq}(x_0, r)}$. Also

$$\sum_{i=0}^{n} \psi^n(d_q(x_0, x_1)) = \sum_{i=0}^{n} \frac{1}{3^n} < 4 = r.$$

We prove that all the conditions of Theorem 2.1 are satisfied. Moreover, S has a fixed point $\frac{1}{2}$.

Theorem 2.8. Let (X, d_q) be a left K-sequentially complete dislocated quasi metric space, $r > 0$, $x_0 \in \overline{B_{dq}(x_0, r)}$ and $S : X \to CP(X)$ be a semi α_*-admissible multifunction on $\overline{B_{dq}(x_0, r)}$. Assume that for $t \in [0, \frac{1}{2})$ such that

$$\alpha_*(Sx, Sy)H_{dq}(Sx, Sy) \leq t(d_q(x, Sx) + d_q(y, Sy)) \; x, y \in \overline{B_{dq}(x_0, r)} \quad (2.4)$$

$$\text{and } \big(dq(x_0, Sx_0)\big) \leq (1 - \theta)r, \quad (2.5)$$

where , $\theta = \frac{t}{1-t}$. If $\{XS(x_0)\}$ is a sequence in $\overline{B_{dq}(x_0, r)}$ and $\{XS(x_0)\} \to x$ and $\alpha(x_n, x_{n+1}) \geq 1$ for $x_n, x_{n+1} \in \{XS(x_0)\}$, $n \in \mathrm{N} \cup \{0\}$. then $\alpha(x_n, x) \geq 1$ or $\alpha(x, x_n) \geq 1$ for all $n \in \mathrm{N} \cup \{0\}$.. Also there exist $x_1 \in Sx_0$ such that $\alpha(x_0, x_1) \geq 1$. Then S has a fixed point in $\overline{B_{dq}(x_0, r)}$.

Proof. As $x_0 \in \overline{B_{dq}(x_0, r)}$ and $S : X \to CP(X)$ be a multivalued mapping on X, then there

exist $x_1 \in Sx_0$ such that $d_q(x_0, Sx_0) = d_q(x_0, x_1)$. If $x_0 = x_1$, then x_0 is a fixed point in $\overline{B_{dq}(x_0, r)}$ of S. Let $x_0 \neq x_1$. From (2.5), we get

$$\big(dq(x_0, Sx_0)\big) \leq (1 - \theta)r < r.$$

It follows that,

$$x_1 \in \overline{B_{dq}(x_0, r)} .$$

Since $\alpha(x_0, x_1) \geq 1$ and S is semi α∗-admissible multifunction on $\overline{B_{dq}(x_0, r)}$, so α∗ $(Sx_0; Sx_1) \geq 1$. Also there exist $x_2 \in Sx_1$ such that $d_q(x_1, Sx_1) = d_q(x_1, x_2)$,

If $x_1 = x_2$, then x_1 is a fixed point of S in $\overline{B_{dq}(x_0, r)}$. Let $x_1 \neq x_2$. Now, we have

$d_q(x_1, x_2) \leq H_{dq}(Sx_0, Sx_1) \leq \alpha *(Sx_0, Sx_1) H_{dq}(Sx_0, Sx_1) \leq t(d_q(x_0, Sx_0) + d_q(x_1, Sx_1))$

$= t(d_q(x_0, x_1) + d_q(x_1, x_2))$.

:

Thus

$$d_q(x_1, x_2) \leq \theta d_q(x_0, x_1),$$

Note that $x_2 \in \overline{B_{dq}(x_0, r)}$ because

$d_q(x_0, x_2) \leq d_q(x_0, x_1) + d_q(x_1,$

$x_2)$

$$\leq d_q(x_0; x_1) + \theta d_q(x_0, x_1),$$
$$\leq (1 + \theta)(1 - \theta) r \leq r$$

As $\alpha * (Sx_0, Sx_1) \geq 1$, $x_1 \in Sx_0$ and $x_2 \in Sx_1$ so $\alpha(x_1, x_2) \geq 1$. As S is semi $\alpha *$ -admissible multifunction on thus $\alpha * (Sx_1, Sx_2) \geq 1$. Let

$x_2,\ldots,x_j \in \overline{B_{dq}(x_0,r)}$ for some $j \in$ N such that $x_{j+1} \in Sx_j$ and $d_q(x_j, Sx_j) = d_q(x_j, x_{j+1})$. As $\alpha*(Sx_0, Sx_1) \geq 1$, we have $\alpha(x_2, x_3) \geq 1$; which further implies

$\alpha*(Sx_2, Sx_3) \geq 1$. Continuing this process, we have $\alpha*(Sx_{j\,1}, Sx_j)\ 1$.

Now, $x_j \in Sx_j$, $x_{j+1} \in Sx_j$, we have $dq(xj, xj+1) = dq(xj, Sxj) \leq Hdq(Sxj\ 1, Sxj)$

$$\leq \alpha*(Sxj\ 1, Sxj)Hdq(Sxj\ 1, Sxj)$$

$$\leq t(d_q(x_{j\,1}, Sx_{j\,1}) + d_q(x_j, Sx_j)$$

$$\leq \theta(d_q(x_{j-1}, x_j)) \leq \ldots \leq \theta^j(d_q(x_0;x_1))$$

Now,

$$dq(x0, xj+1) \leq$$

$$d_q(x_0,x_1) + d_q(x_1,x_2) + \ldots + d_q(x_j,x_{j+1})$$
$$d_q(x_0,x_1) + \theta d_q(x_0,x_1) + \ldots + \theta^j(d_q(x_0,x_1))$$
$$(1-\theta)r\frac{(1-\theta^{j+1})}{(1-\theta)} \leq r.$$

Thus $x_{j+1} \in \overline{B_{dq}(x_0,r)}$. As $\alpha*(Sx_{j\,1};Sx_j) \geq 1$, $x_j \in Sx_{j-1}$, $x_{j+1} \in Sx_j$, we have $\alpha(x_j;x_{j+1}) \geq 1$.

Also S is semi $\alpha*$-admissible multifunction on $\overline{B_{dq}(x_0,r)}$, therefore $\alpha*(Sx_j, Sx_{j+1}) \geq 1$. Hence by mathematical induction, $x_n \in \overline{B_{dq}(x_0,r)}$ and $\alpha*(Sx_n, Sx_{n+1}) \geq 1$ for all $n \in$ N. Now,

$$d_q(x_n;x_{n+1}) \leq \theta^n(d_q(x_0;x_1)),\ \text{for all } n \in \mathrm{N}.$$

Now,

$$dq(xn, xn+i) \leq \begin{array}{l} d_q(x_n, x_{n+1}) + ... + d_q(x_{n+i-1}, x_{n+i}) \\ \dfrac{\theta^n(1-\theta^i)}{1-\theta} d_q(x_0, x_1) \longrightarrow 0 \text{ as } n \to \infty \end{array}$$

Thus we proved that $\{x_n\}$ is a Cauchy sequence in ($\overline{B_{dq}(x_0, r)}$, d_q). As every closed ball in a left *K*-sequentially complete dislocated quasi metric space is left *K*-sequentially complete, so there exists $x^* \in \overline{B_{dq}(x_0, r)}$ such that $x_n \to x^*$, and

$$lim_{n\to\infty}\ d_q(x_n, x\ ) = lim_{n\to\infty}\ d_q(x\ , x_n)$$

Note that $\{x_n\}$ is a $\{XS(x_0)\}$ in $\overline{B(d_q(x_0, r))}$. As $\alpha * (Sx_n, Sx_{n+1}) \geq 1$ for all $n \in$ N, we have $\alpha(xn+1;xn+2) \geq 1$ for all $n \in$ N∪ {0}. By assumption, we have $\alpha(x_n, x\ ) \geq 1$ for all $n \in$ N∪ {0}.

Thus $\alpha * (Sx_n, Sx\ ) \geq 1$. Now

$d_q(x\ , Sx\ ) \leq d_q(x\ , x_{n+1}) + d_q(x_{n+1}, Sx\ )$

$\leq d_q(x\ , x_{n+1}) + H_{dq}(Sx_n, Sx\ )$

$\leq d_q(x\ , x_{n+1}) + \alpha * (Sx_n, Sx_{n+1}) H_{dq}(Sx_n, Sx\ )$

$\leq d_q(x\ , x_{n+1}) + t(d_q(x_n; x_{n+1}) + d_q(x\ , Sx\ ))$.

Letting $n \to \infty$, we obtain $d_q(x\ , Sx\ ) = 0$: Similarly, if $\alpha(x\ , x_n) \geq 1$ for all $n \in$ N∪ {0}.

We obtain $d_q(Sx\ ,\ x\ ) = 0$. Hence $x \in Sx$. So S has a fixed point in $\overline{B(d_q(x_0, r))}$.

Corollary 2.9. Let $(X, \leq, d_q)$ be a left K-sequentially complete dislocated quasi metric space, $S : X \to CP(X)$. Suppose there exists $t \in [0, \frac{1}{2})$ with $H_{dq}(Sx;Sy) \leq t(d_q(x;Sx)+d_q(y;Sy))$ for all elements x,y in $\overline{B(d_q(x_0, r))}$ with $x \leq y$,

$$\left(dq(x_0, Sx_0)\right) \leq (1-\theta)\text{r}.$$

for $x_0 \in \overline{B_{dq}(x_0, r)}$, $n \in \mathbb{N}$, $r > 0$, $\theta = \frac{t}{1-t}$. If $\{XS(x_0)\}$ is a sequence in $\overline{B_{dq}(x_0, r)}$ and $\{XS(x_0)\} \to x$ and $x_n \leq x_{n+1}$ for $x_n, x_{n+1} \in \{XS(x_0)\}$, then $x \leq x_n$ or $x_n \leq x$, for all $n \in \mathbb{N} \cup \{0\}$. Also there exist $x_1 \in Sx_0$ such that $x_0 \leq x_1$. If $x, y \in \overline{B_{dq}(x_0, r)}$ such that $x \leq y$ implies $Sx \leq_r Sy$. Then there exists a point x in $\overline{B_{dq}(x_0, r)}$ such that $x \in Sx$.

3 Fixed point results for graphic contractions

Consistent with Jachymski [39], let (X, d) be a metric space and denotes the diagonal of the Cartesian product $X \times X$. Consider a directed graph G such that the set $V(G)$ of its vertices coincides with X, and the set $E(G)$ of its edges contains all loops, i.e., $E(G)$. We assume G has no parallel edges, so we can identify G with the pair $(V(G), E(G))$. Moreover, we may treat G as a weighted graph (see [39]) by assigning to each edge the distance between its vertices. If x and y are vertices in a graph G, then a path in G from x to y of length N

($N \in$ N) is a sequence $\{x_i\}_{i=0}^{N}$ of $N+1$ vertices such that $x_0 = x$, $x_N = y$ and $(x_{n-1}, x_n) \in E(G)$ for $i = 1,...,N$. A graph G is connected if there is a path between any two vertices. G is weakly connected if $\tilde{G}$ is connected (see for details [12, 13, 21, 39]).

Definition 3.1[39]. We say that a mapping $T : X \to X$ is a Banach G-contraction or simply G-contraction if T preserves edges of G, i.e.,

$$\forall x,y \in X((x, y) \in E(G)\ )\ (Tx, Ty) \in E(G))$$

and T decreases weights of edges of G in the following way:

$$\exists k \in (0,1)\ \forall x,y \in X((x, y) \in E(G)\ ) \mapsto d(Tx, Ty) \leq kd(x, y)).$$

Definition 3.2. Let (X, d_q) be a dislocated quasi metric space endowed with a graph G and $S: X \to CP(X)$ be closed proximal mapping. Assume that for $r > 0$, $x_0 \in X$ and $\psi \in \Psi$, following conditions hold,

$$\forall x,y \in \overline{B(d_q(x_0, r))}\ ((x, y) \in E(G)\ )\ (Sx, Sy) \in E(G))$$

$$\forall x,y \in \overline{B(d_q(x_0, r))}\ ((x, y) \in E(G)\ ) \mapsto d_q(Sx,Sy) \leq \psi\ (M_q(x, y))$$

where

$$M_q(x, y) = \max\{d_q(x;y), d_q(x,Tx), d_q(y,Ty)\}.$$

Then the mapping S is called a ψ-graphic contractive mapping. If $(t) = kt$ for some $k \in [0,1)$, then we say S is G-contractive mappings.

In this section, we give fixed point results on a dislocated quasi metric space endowed with a graph.

Theorem 3.3. Let (X, d) be a left K-sequentially complete dislocated quasi metric space endowed with a graph G and $S: X \to CP(X)$. Suppose the following assertions hold:

(i) if $\forall x,y \in \overline{B(d_q(x_0,r))}$ $((x, y) \in E(G))$ $(Sx, Sy) \in E(G))$;

(ii) there exists $x_0 \in \overline{B(d_q(x_0,r))}$, $x_1 \in Sx_0$ such that $(x_0, x_1) \in E(G)$ and $d_q(x_0, Sx_0) \leq (1-\lambda)r$, such that $H_{dq}(Sx, Sy) \leq \max\{d_q(x;y), d_q(x,Tx), d_q(y,Ty)\}$, for all $x, y \in \overline{B_{dq}(x_0,r)}$, $(x, y) \in E(G)$;

(iii) . If $\{XS(x_0)\}$ is a sequence in $\overline{B_{dq}(x_0,r)}$ and $\{XS(x_0)\} \to x$ and $(x_n, x_{n+1}) \in E(G)$ for $x_n, x_{n+1} \in \{XS(x_0)\}$, $n \in \mathrm{N} \cup \{0\}$, then $(x_n, x) \in E(G)$ or $(x, x_n) \in E(G)$ for all $n \in \mathrm{N} \cup \{0\}$.

Then S has a fixed point in $\overline{B_{dq}(x_0,r)}$,.

Proof. Define $\alpha: X \times X \to [0;+\infty)$ by

$$\alpha(x, y) = 1 \; if \; (x, y) \in E(G)1, \text{ otherwise } 0.$$

At first we prove that S is a semi $\alpha*$-admissible multifunction on $\overline{B_{dq}(x_0,r)}$. Let $\alpha(x, y) \geq 1$, then $(x, y) \in E(G)$. From (i), we have, $(Sx, Sy) \in E(G)$. That is, $\alpha*(Sx,Sy) \geq 1$. Thus S is a semi$\alpha*$-admissible multifunction on $\overline{B_{dq}(x_0,r)}$.

From (ii) there exists $x_0 \in \overline{B(d_q(x_0,r))}$, $x_1 \in Sx_0$ such that $(x_0, x_1) \in E(G)$ and $d_q(x_0, Sx_0) \leq (1-\lambda)r$.

. That is, α (x_0, x_1))≥1,   and $d_q(x_0, Sx_0) \leq (1-\lambda)r$.

If $(x, y) \in E(G)$, then $(Sx, Sy) \in E(G)$and hence α $(Sx, Sy) = 1$. Thus, from (ii) we have

(iv) α (Sx, Sy) $H_{dq}(Sx, Sy)$ = $H_{dq}(Sx, Sy) \leq$ $\max\{d_q(x;y), d_q(x,Tx), d_q(y,Ty)\}$, for all $x, y \in \overline{B_{dq}(x_0,r)}$. Condition ($iii$) implies that, If $\{XS(x_0)\}$ is a sequence in $\overline{B_{dq}(x_0,r)}$ and $\{XS(x_0)\} \to x$ and α $(x_n, x_{n+1}) \geq 1$ for $x_n, x_{n+1} \in \{XS(x_0)\}$, $n \in \mathrm{N} \cup \{0\}$, then $\alpha(x_n,x) \geq 1$ or $\alpha(x, x_n) \geq 1$ for all $n \in \mathrm{N} \cup \{0\}$.

Hence, all conditions of Theorem 2.1 are satified. Hence S has a fixed point in $\overline{B_{dq}(x_0,r)}$.

Corollary 3.4. Let (X, d) be a left K-sequentially complete dislocated quasi metric space endowed with a graph G and S: $X \to CP(X)$. Suppose the following assertions hold:

(i) If $\forall x,y \in \overline{B(d_q(x_0,r))}$ $((x, y) \in E(G))$ $(Sx, Sy) \in E(G))$;

(ii) there exists $x_0 \in \overline{B(d_q(x_0,r))}$, $x_1 \in Sx_0$ such that $(x_0, x_1) \in E(G)$ and $d_q(x_0, Sx_0) \leq (1-\lambda)r$;

(iii) there exists t∈ $[0, \frac{1}{2})$ such that $H_{dq}(Sx, Sy) \leq t[d_q(x;Sx) + d_q(y;Sy)]$for all $x, y \in$ $\overline{B_{dq}(x_0,r)}$, $((x, y) \in E(G)$, where $\theta = \frac{t}{1-t}$.

(i) if $\{XS(x_0)\}$ is a sequence in $\overline{B_{dq}(x_0,r)}$ and $\{XS(x_0)\} \to x$ and $(x_n, x_{n+1}) \in E(G)$ for $x_n, x_{n+1} \in \{XS(x_0)\}$, $n \in \mathrm{N} \cup \{0\}$, then $(x_n, x) \in E(G)$ or $(x, x_n) \in E(G)$ for all $n \in \mathrm{N} \cup \{0\}$.

Then S has a fixed point in $\overline{B_{dq}(x_0,r)}$.

Corollary 3.5. Let (X, d) be a left K-sequentially complete dislocated quasi metric space endowed with a graph G and S: $X \to CP(X)$. Suppose the following assertions hold:

(i) If $\forall x,y \in \overline{B(d_q(x_0,r))}$ $((x, y) \in E(G))$ $(Sx, Sy) \in E(G))$;

(ii) there exists $x_0 \in \overline{B(d_q(x_0,r))}$ such that $(x_0, Sx_0) \in E(G)$ and $\sum_{i=0}^{n} k^i d_q(x_0, Sx_0) \leq r$, for all $n \in \mathbb{N} \cup \{0\}$.

(iii) for all $x,y \in \overline{B(d_q(x_0,r))}$, $(x, y) \in E(G)$, $H_{dq}(Sx, Sy) \leq k\, d_q(x,y)$, where $k \in [0,1)$;

(iv) if $\{XS(x_0)\}$ is a sequence in $\overline{B_{dq}(x_0,r)}$ and $\{XS(x_0)\} \to x$ and $(x_n, x_{n+1}) \in E(G)$ for $x_n, x_{n+1} \in \{XS(x_0)\}$, $n \in \mathbb{N} \cup \{0\}$, then $(x_n, x) \in E(G)$ or $(x, x_n) \in E(G)$ for all $n \in \mathbb{N} \cup \{0\}$.

Then S has a fixed point in $\overline{B_{dq}(x_0,r)}$.

4 Application

In this section, we discuss an application of fixed point theorem we proved in the previous section in solving the system of Volterra type integral equation. Such system is given by the following equation:

$$u(t) = \int_0^t K(t,s,u(s))ds + f(t); \qquad (4.1)$$

for $t \in [0, a]$, where $a > 0$. We find the solution of the system (4.1). Let $C([0, a], \mathrm{R})$ be the space of all continuous functions defined on $[0, a]$. For $u \in C([0, a], \mathrm{R})$

define supremum norm as: $\|u\| = \sup_{t\in[0,a]} \{u(t)\}$. Let $C([0, a], \mathrm{R})$ be endowed with the metric

$$d(u,v) = \sup_{t\in[0,a]} \| \, |u(t) - v(t)| \, \| \qquad (4.2)$$

for all $u,v \in C([0, a], \mathrm{R})$. With these setting $C([0, a], \mathrm{R}), \|.\|)$ becomes Banach

space.

Now we prove the following theorem to ensure the existence of solution of system of integral equations. For more details on such applications we refer the reader to [9, 28].

Theorem 4.1. Assume the following conditions are satisfied:

(i) $K : [0, a]\times[0, a] \times \mathrm{R} \to \mathrm{R}$ and $f : [0, a] \to \mathrm{R}$ are continuous;

(ii) Define

$$Su(t) = \int_0^t K(t, s, u(s))ds + f(t)$$

Suppose $\propto, \boldsymbol{\kappa}$: $[0, a]\times[0, a] \to \mathrm{R}^+$, there exist $\lambda \in (01)$, such that

$$|K(t,s,u) - K(t,s,v)| \leq [M(u, v)]$$

for all $t, s \in [0, a]$ and $u,v \in C([0, a], \mathrm{R})$, and

$$\left| \int_0^t \kappa(x,y) \right| \leq \lambda t$$

where

$M(u, v)= \max\{|Su(t) - Sv(t)|, |u(t) - Su(t)|, |v(t) - Sv(t)|\}.$

Then the system of integral equations given in (4.1) has a solution.

Proof. Define the mapping $\propto: [0, a]\times[0, a] \to R^+$ by

$\alpha(u, v) = 1$ *if* $u, v \in [0, a]$, otherwise 0.

Take $(t) = t$. By assumption

$$\begin{aligned} |Su(t) - Sv(t)| &= \int_0^t |K(t, s, u(s) - K(t, s, v(s)))|\, ds \\ &\leq \int_0^t \kappa(x, y)([M(u, v)])ds \\ &\leq \left|\int_0^t \kappa(x, y)\right| \|M(u, v)\| ds \\ &\leq \lambda \|M(u, v)\|. \end{aligned}$$

This implies

$$|Su(t) - Sv(t)| \leq \lambda \|M(u, v)\|,$$

That is

$$\alpha(u, v)\|Su(t) - Sv(t)\| \leq \psi \|M(u, v)\|$$

So all the conditions of Theorem 2.1 are satisfied. Hence the system of integral equations given in (4.1) has a unique solution. ■

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