

## **Super Resolution Based Image Compression Using Wavelet Transforms and Singular Value Decomposition**

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**Abstract:** In this paper we propose a new lossy image compression technique which uses Singular value decomposition (SVD) and wavelet difference reduction (WDR) technique followed by resolution enhancement using discrete wavelet transform (DWT) and stationary wavelet transform (SWT). The input image is decomposed into four different frequency subbands by using DWT. The low frequency subband is the being compressed by using DWR and in parallel the high frequency subbands are being compressed by using SVD which reduces the rank by ignoring small singular values. The compression ratio is obtained by dividing the total number of bits required to represent the input image over the total bit numbers obtain by WDR and SVD. Reconstruction is carried out by using inverse of WDR to obtained low frequency subband and reconstructing the high frequency subbands by using matrix multiplications. The high frequency subbands are being enhanced by incorporating the high frequency subbands obtained by applying SWT on the reconstructed low frequency subband. The reconstructed low frequency subband and enhanced high frequency subbands are being used to generate the reconstructed image by using inverse DWT. The visual and quantitative experimental results of the proposed image compression technique are shown and also compared with those of the WDR with arithmetic coding technique and JPEG2000. From the results of the comparison, the proposed image compression technique outperforms the WDR-AC and JPEG2000 techniques.

**Keywords:** lossy image compression, singular value decomposition, wavelet difference reduction, stationary wavelet transform, discrete wavelet transform, image super resolution.

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### **INTRODUCTION**

With the growing demand for multimedia applications especially high definition images, efficient storage and transmission of images have been issues of great concern [1-4]. Image processing deals with the reduction of the amount of bits used to represent an image. Not only that but also resolution of an image plays an important role in many image processing applications,

such as video resolution enhancement [5], feature extraction [6], and satellite image resolution enhancement [7]. Hence compression of an image and yet reconstruct the image with good resolution is important.

There are two categories of image compression techniques namely lossless and lossy image compression techniques [8, 9]. In lossless image compression, the original image can be perfectly recovered from the compressed image while in lossy compression the original image cannot be perfectly recovered from the compressed image because some information is lost as a result of compression. Lossless compression is used in applications with high requirements such as medical imaging. Lossy compression techniques are very popular because they offer higher compression ratio. The objective of image compression is to achieve as much compression as possible with little loss of information [10, 11].

Wavelets are also playing significant role in many image processing applications [12-15]. The two-dimensional wavelet decomposition of an image is performed by applying the one-dimensional DWT along the rows of the image first, and then the results are decomposed along the columns. This operation results in four decomposed subband images referred to Low-Low (LL), Low-High (LH), High-Low (HL), and High-High (HH).

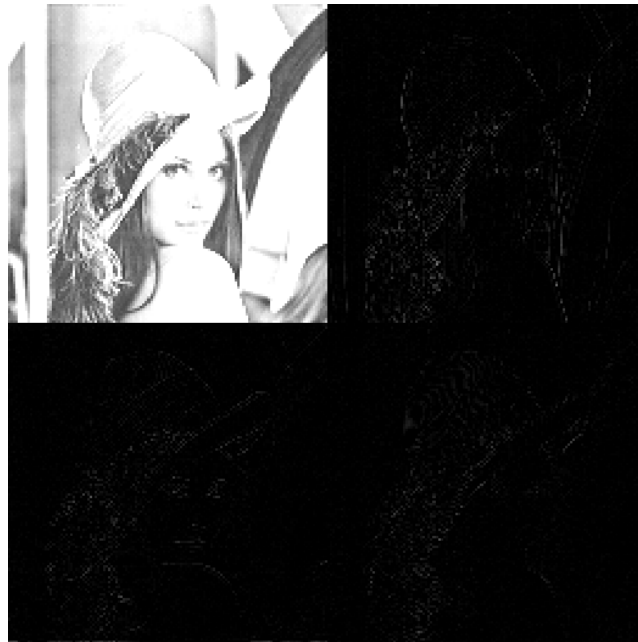


Fig. 1: LL, LH, HL, and HH subbands of Lena image obtained by using DWT.

The frequency components of those subbands cover the full frequency spectrum of the original image. Fig. 1 shows different subband images of Lena image where the top left image is the LL subband and the bottom right image is the HH subband.

In this research work, a new lossy compression technique which employs Singular value decomposition (SVD) and wavelet difference reduction (WDR) is presented. SVD is a lossy image compression technique which can be regarded as a quantization process where it reduces the psychovisual redundancies of the image [16, 17]. In order to enhance the resolution of the decompressed image stationary wavelet transform (SWT) is used. WDR is one of the state-of-the-art techniques in image compression which uses wavelet transform. It is a lossy image compression technique which achieves compression by first taking the wavelet transform of the input image and then applying the difference reduction method on the transform values [18-21].

In the proposed compression technique, the input image is firstly decomposed into its different frequency subbands by using 1 level DWT. The LL subband is then being compressed by using DWR and the high frequency subbands, i.e. LH, HL, and HH, are being compressed by using SVD. The proposed technique has been tested on several well-known images such as, Lena, Peppers, Boat, and Airfield. The results of this technique have been compared with those of JPEG2000 and WDR with arithmetic coding techniques. The quantitative experimental results based on PSNR show that the proposed technique overcomes the aforementioned techniques. The SVD and WDR image compression techniques are discussed in the next section.

**REVIEW OF SINGULAR VALUE DECOMPOSITION AND WAVELET DIFFERENCE REDUCTION**

An image is represented by a matrix. SVD is a technique used to decompose a given matrix  $A$ , into three matrices  $U$ ,  $\Sigma$ , and  $V$ .  $U$  and  $V$  are orthogonal while  $\Sigma$  is a diagonal matrix containing the singular values  $A$ . The following equations summarize the SVD procedure. Eqn. (1) shows the sizes of the matrices  $U$ ,  $\Sigma$ , and  $V$  for a given matrix  $A$  with size  $m \times n$  [22-24].

$$A_{m \times n} = U_{m \times m} \Sigma_{m \times n} (V_{n \times n})^T \tag{1}$$

Eqn. (2) shows how a smaller rank is used to approximate the diagonal matrix  $\Sigma$ .

$$\Sigma_{m \times n} = \begin{bmatrix} \bar{\Sigma}_{p \times q} & 0 \\ 0 & 0 \end{bmatrix} \quad p \leq m \text{ and } q \leq n \tag{2}$$

Some columns of  $U$  and rows of  $V$  are then reduced in order to reconstruct the compressed image by multiplication. This is shown in eqn. (3)

$$U_{m \times m} = \begin{bmatrix} \bar{U}_{m \times p} & U_{m \times (m-p)} \end{bmatrix} \tag{3}$$

$$V_{n \times n} = \begin{bmatrix} \bar{V}_{n \times q} & V_{n \times (n-q)} \end{bmatrix}$$

The compressed image is then obtained as shown in eqn. (4)

$$A_{m \times n} = \bar{U}_{m \times p} \bar{\Sigma}_{p \times q} (\bar{V}_{n \times q})^T \tag{4}$$

Because the singular matrix has sorted singular values (in descending order) by using the physcovisual concept, ignoring low singular value will not significantly reduce the visual quality of the image. Fig. 2 is showing the Lena’s picture being reconstructed by using different amount of singular values. This characteristic that an image can be reconstructed by fewer amounts of singular values takes SVD suitable for compression. Because after reconstruction of the image the ignored singular values cannot be recovered, the compression by SVD is lossy [24].

The WDR is a compression technique based on the difference reduction method. The wavelet transform of the input image is first taken; bit plane encoding is then applied to the transform values. The bit plane encoding procedure starts with the initialization stage, where a threshold  $T_o$  is chosen such that  $T_o$  is greater than all the transform values and at least one of the transform values has a magnitude of  $T_o/2$ . The next stage is the initialization stage where the threshold  $T = T_{k-1}$  is updated to  $T = Tk$ , where  $T_k = T_{k-1}/2$ . New significant transform values ( $w(i)$ ) which are satisfying  $T \leq |w(i)| \leq 2T$  are then identified at the significant pass stage. The transform values of these significant transform values are then encoded using the difference reduction method. At the significant pass stage, already quantized values ( $w_Q$ ) which satisfy  $|w_Q| \geq 2T$  are then refined in order to reduce error [19-21].



Fig. 2: Lena's image of size 256x256 reconstructed by eq. (4) (a) original Lena image (b)reconstructed using 128  $\sigma$  (c) reconstructed using 64  $\sigma$  (d) reconstructed using 32  $\sigma$

### THE PROPOSED LOSSY IMAGE COMPRESSION TECHNIQUE

The proposed image compression technique is a lossy compression technique. Firstly, the image is decomposed into its frequency subbands by using DWT. Among these subbands, LL subband is being compressed by using WDR. The high frequency subband images are being compressed by using SVD. The number of singular values that are being used in order to reconstruct the high frequency subbands can be reduced into 1, i.e. the highest singular value is enough to reconstruct the high frequency subbands. If only one singular value is being used in order to reconstruct a matrix this means that only one column of U and V matrices are being used. The qualitative loss is not psychovisually noticeable up to some point. In order to obtain the compression ratio of the proposed technique, the total number of bits required to represent the original image is divided by the total of number of bits which is obtain by adding the number of bit streams of WDR for LL and that of the SVD compression for LH, HL, and HH.

Decompression is carried out by taking the inverse WDR (IWDR) of the bit streams in order to reconstruct the LL subband and in parallel the matrix multiplications are conducted in order to reconstruct LH, HL, and HH subbands. Due to the losses by ignoring low-valued singular values, high frequency subbands need to be enhanced. For this purpose, stationary wavelet transform (SWT) is applied to the LL subband image which results into new low and high frequency subbands. These high frequency subbands will have the same direction as the ones obtained by DWT (e.g. horizontal, vertical, and diagonal), so they will be added to the respective

ones reconstructed by matrix multiplications. Now, the LL subband image obtained by IWDR and the enhanced LH, HL, and HH subbands are combined by using inverse DWT (IDWT) in order to reconstruct the decompressed image. The enhancement of high frequency subbands by using SWT results into more sharpen decompressed image. The block diagram of the proposed lossy image compression technique is shown in Fig. 3. The experimental qualitative and quantitative results are represented and discussed in the next section.

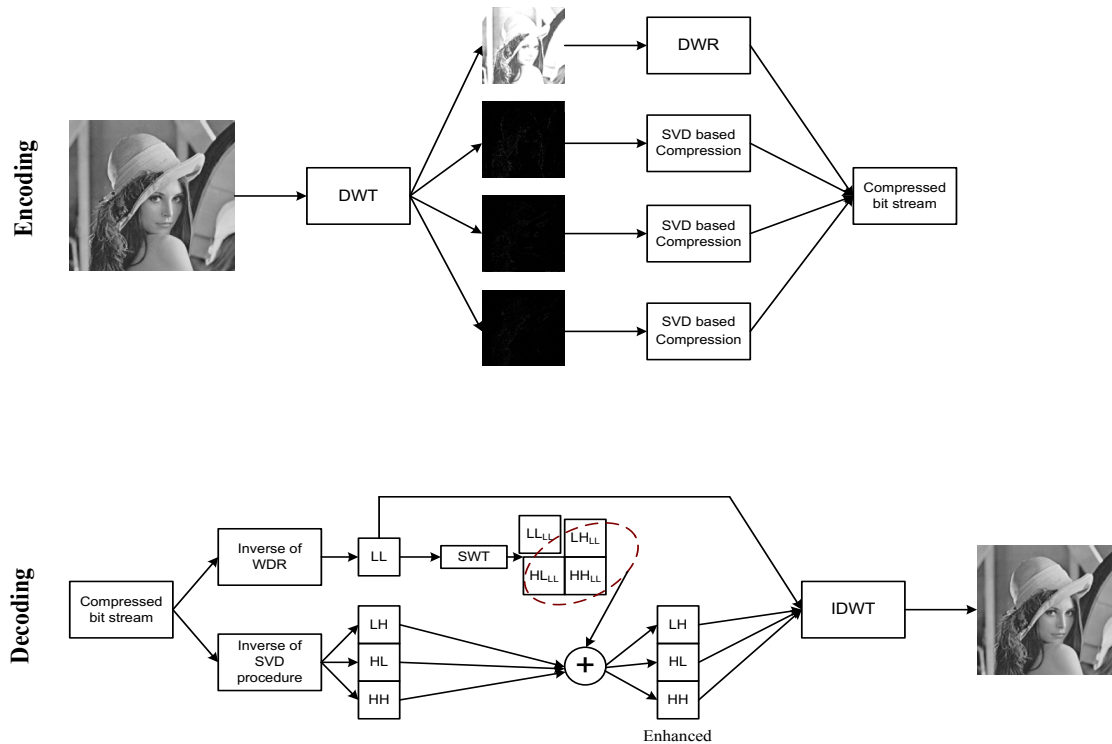


Fig. 3: The block diagram of the proposed blocked based lossy image compression technique.

**EXPERIMENTAL RESULTS AND DISCUSSION**

As it was mentioned in the Introduction, for comparison purposes the proposed lossy image compression was tested on Airfield, Boats, Lena, and Peppers images. All the input images are 256x256, 8-bit gray scale. Table 1, 2, and 3 are showing the quantitative comparison between the proposed techniques and JPEG2000 and WDR [25-26] by use of PSNR in dB for compression ratio of 20:1, 40:1, and 80:1 respectively.

Table 1: PSNR values in dB for 20:1 compression

Image	Techniques		
	WDR	JPEG2000	<b>Proposed</b>
Lena	35.72	35.99	<b>39.14</b>
Pepper	34.21	35.07	<b>40.07</b>
Boats	32.42	33.18	<b>35.97</b>
Airfield	27.02	27.32	<b>31.43</b>
Goldhill	31.76	32.18	<b>38.05</b>

Table 2: PSNR values in dB for 40:1 compression

Image	Techniques		
	WDR	JPEG2000	<b>Proposed</b>
Lena	32.44	32.75	<b>35.98</b>
Pepper	31.67	32.40	<b>36.45</b>
Boats	29.32	29.76	<b>32.03</b>
Airfield	24.72	24.88	<b>29.62</b>
Goldhill	29.43	29.72	<b>34.19</b>

Table 3: PSNR values in dB for 80:1 compression

Image	Techniques		
	WDR	JPEG2000	<b>Proposed</b>
Lena	29.71	29.62	<b>32.46</b>
Pepper	28.93	29.54	<b>33.07</b>
Boats	26.96	26.76	<b>30.19</b>
Airfield	22.71	22.64	<b>27.32</b>
Goldhill	27.72	27.69	<b>32.64</b>

As the PSNR values shows, the performance of the proposed technique overcomes the JPEG2000 and WDR based image compression techniques. Fig. 4 is showing a portion of the magnified Lena image being compressed with compression ratio of 40:1 by using JPEG2000, WDR, and the proposed image compression technique.

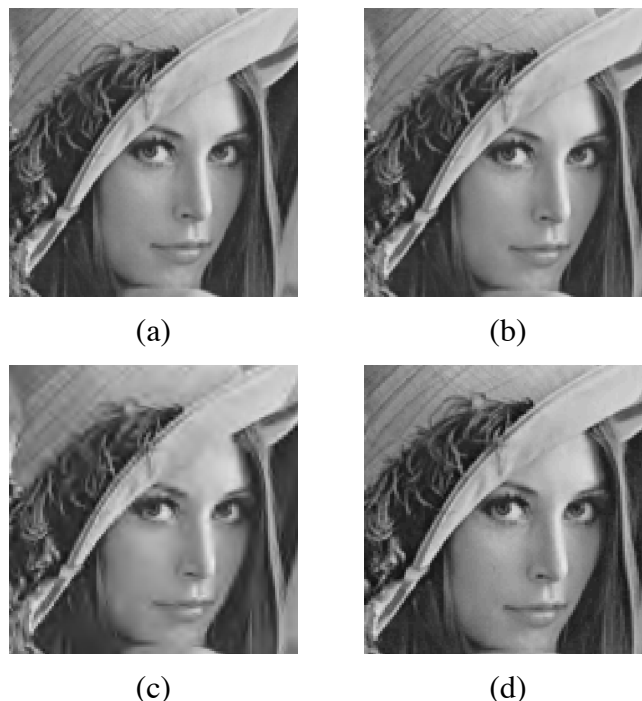


Fig. 4: Zoomed (a) original Lena image, and compressed images by using (b) JPEG2000, (c) WDR, and (d) the proposed image compression technique at 40:1 compression ratio

**CONCLUSION**

In this research work a new lossy image compression technique which uses SVD and WDR technique followed by resolution enhancement using DWT and SWT was proposed. The input

image was decomposed into four different frequency subbands by using DWT. The low frequency subband was the being compressed by using DWR and in parallel the high frequency subbands were being compressed by using SVD. The compression ratio was obtained by dividing the total number of bits required to represent the input image over the total bit numbers obtain by WDR and SVD. Reconstruction was carried out by using inverse of WDR to obtained low frequency subband and reconstructing the high frequency subbands by using matrix multiplications. The high frequency subbands were being enhanced by using high frequency obtained by SWT. The reconstructed low frequency subband and enhanced high frequency subbands were being used to generate the reconstructed image by using inverse DWT. The visual and quantitative experimental results of the proposed image compression technique were showing that the proposed image compression technique was outperforming the WDR and JPEG2000 techniques.

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